Special Relativistic Analogues of Black Strings??

The Dark Side of Extra Dimensions
BIRS Meeting
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THANKS TO
THE ORGANIZERS
BIRS, UofA/TPI, CIAR, CITA,
PIMS, PITP

Are there special relativistic analogues of black strings?

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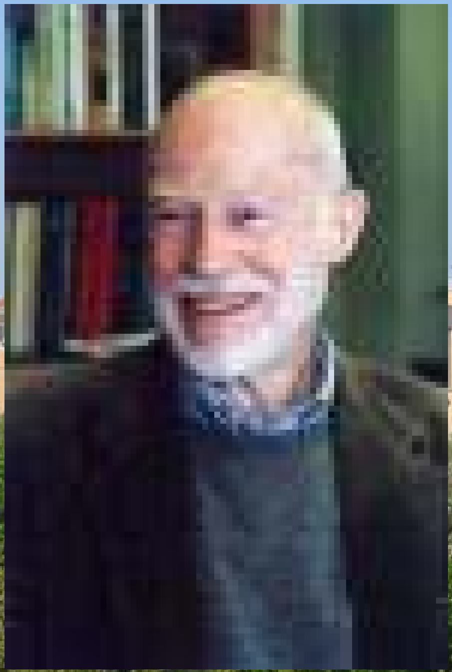
I don't know yet!

Dedication

Bryce DeWitt (1923-2004)



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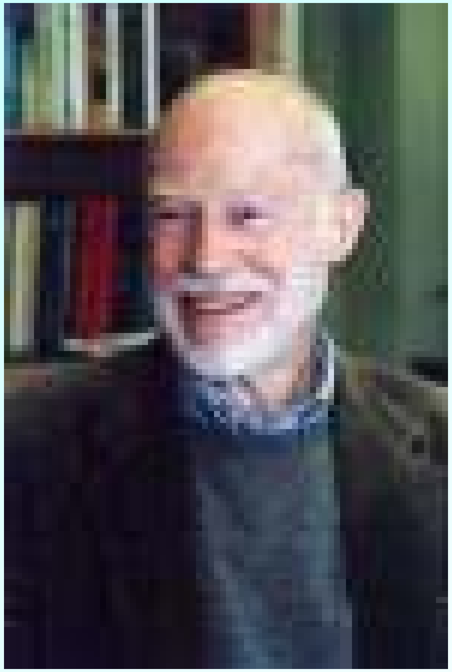

"They're only a set of PDE's Larry! Go solve them!"

Co-conspirators (perhaps unwitting)

- Luis Lehner (LSU)
- Frans Pretorius (Caltech)
- Inaki Olabarrieta (somewhere in the Basque country)
- Roman Petryk (UBC)
- Hugo Villegas (somewhere in Mexico)
- David Garfinkle (Oakland U)

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YOU!!

i.e. audience participation encouraged!!

Gregory & Laflamme Instability

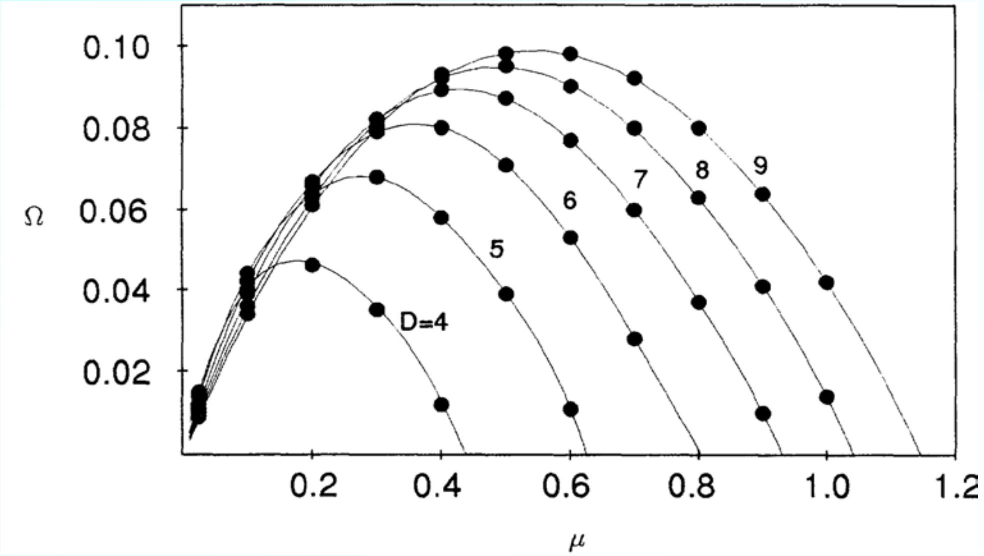
[Gregory & Laflamme, PRL, 70, 2837, (1993)]

- 5D Black string (Myers & Perry, Ann Phys, 172, 304 (1986))

$$ds^2 = -\left(1 - \frac{R}{2M}\right) dT^2 + \left(1 - \frac{R}{2M}\right)^{-1} dR^2 + R^2 d\Omega^2 + dZ^2$$

infinite: $-\infty < Z < +\infty$ periodic: $0 \le Z < L$; $Z = 0, Z = L$ identified

- Gregory & Laflamme found that on this background (among others) perturbations of sufficiently long wavelength were unstable, noted that collapse to 5D black hole was entropically favoured, and that such collapse would seem to imply violation of cosmic censorship.



- Critical length: $L_C \approx 14.3M$

What do we know about the end state of Gregory-Laflamme-unstable black strings?

[Choptuik et al PRD, 68, 044001 (2003)]

- Adopt "4+1" (2+2+1) approach
- Spacetime coordinates

$$x^\mu = (t, r, \theta, \varphi, z) \equiv (t, x^A, x^\Omega)$$

- Retain spherical symmetry w.r.t r in angular variables, all fcns then

$$f(t, r, z)$$

- Metric

$$ds^2 = (-\alpha^2 + \gamma_{AB}\beta^A\beta^B)dt^2 + 2\gamma_{AB}\beta^A dx^B dt + \gamma_{AB}dx^A dx^B + \gamma_\Omega d\Omega^2$$

$$\alpha = \alpha(t, r, z) \quad \beta^A = \begin{bmatrix} \beta^r(t, r, z) \\ \beta^z(t, r, z) \end{bmatrix} \quad \gamma_{AB} = \begin{bmatrix} \gamma_{rr}(t, r, z) & \gamma_{rz}(t, r, z) \\ \gamma_{rz} & \gamma_{rr}(t, r, z) \end{bmatrix} \quad \gamma_\Omega = \gamma_\Omega(t, r, z)$$

What do we know about the end state of Gregory-Laflamme-unstable black strings?

- Coordinate conditions (3 degrees of freedom): based on ingoing-null Eddington-Finkelstein coordinates for the static black string (metric smooth, nicely behaved across horizon)

$$ds_{BS}{}^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + \frac{4M}{r}drdt + \left(1 + \frac{2M}{r}\right)^{-1}dr^2 + r^2 d\Omega^2 + dz^2$$

- t coordinate (slicing condition)

$$\alpha = \alpha_{BS} = \frac{1}{\sqrt{1 + \frac{2M}{r}}}$$

- r coordinate chosen so that $g_{\theta\theta}$ remains constant during evolution

$$\beta^r = \frac{2\alpha K_{\theta\theta}}{\gamma_{\theta\theta,r}}$$

- z coordinate

$$\beta^z = \beta^z_{BS} = 0$$

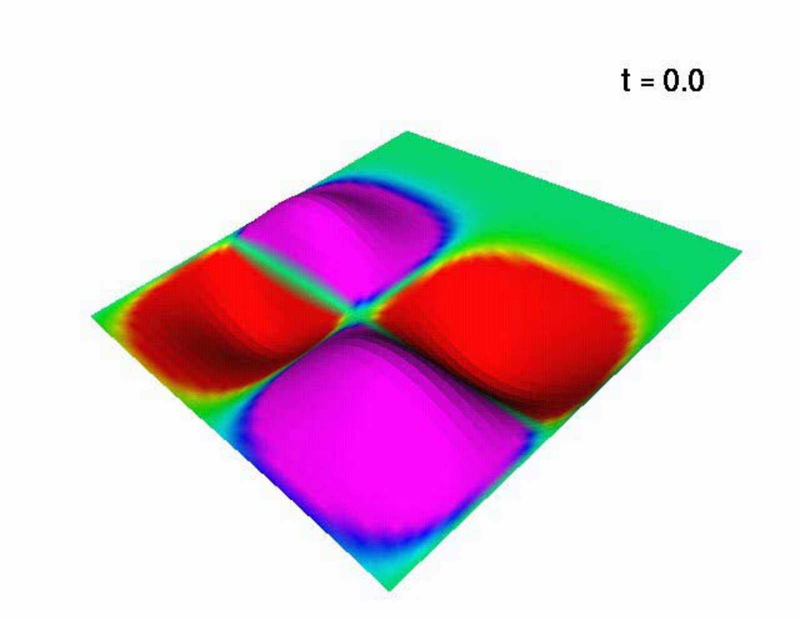
Implementation Details
(Most of relevance to, e.g., binary black hole problem)

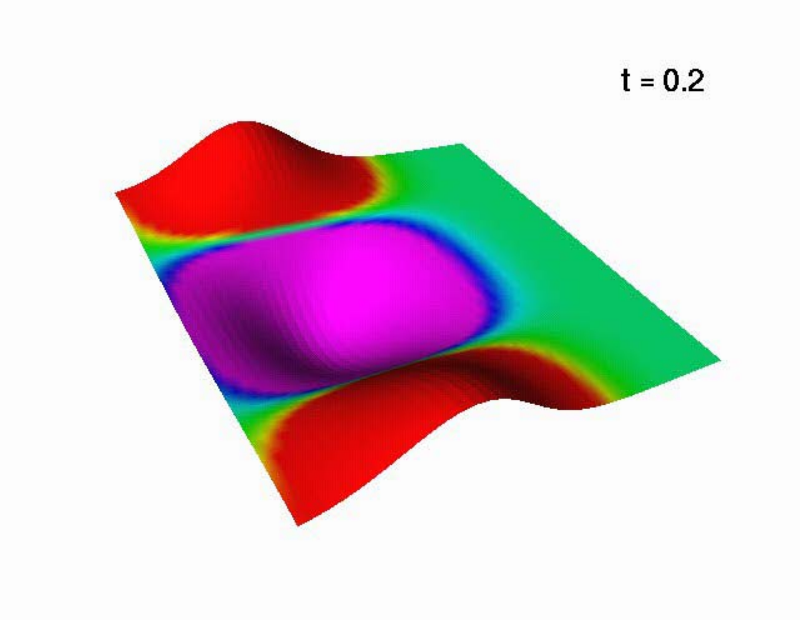
- Black hole excision used
- Spatial domain compactified in a straightforward fashion
- Finite difference approach is Crank-Nicholson (solved iteratively) with $O(h^2)$ spatial differences and $O(h^4)$ Kreiss-Oliger dissipation
- Computation done on large Beowulf clusters, longest single runs took of order a week on of order 100 processors (and this is a "2D" problem!!)

What do we know about the end state of Gregory-Laflamme-unstable black strings?

[Choptuik et al PRD, 68, 044001 (2003)]

TIME DEVELOPMENT OF METRIC COMPONENTS: STABLE STRING



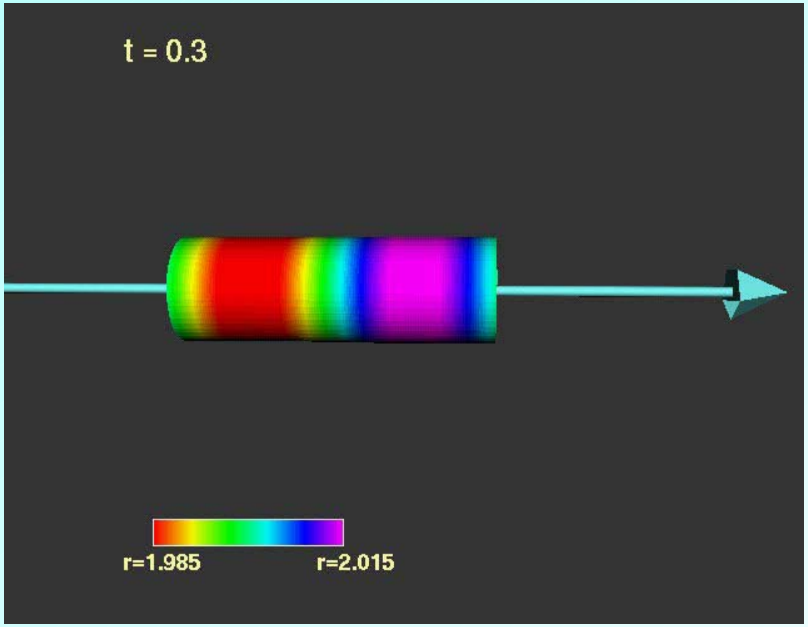


$g_{tt}(t,r,z)$

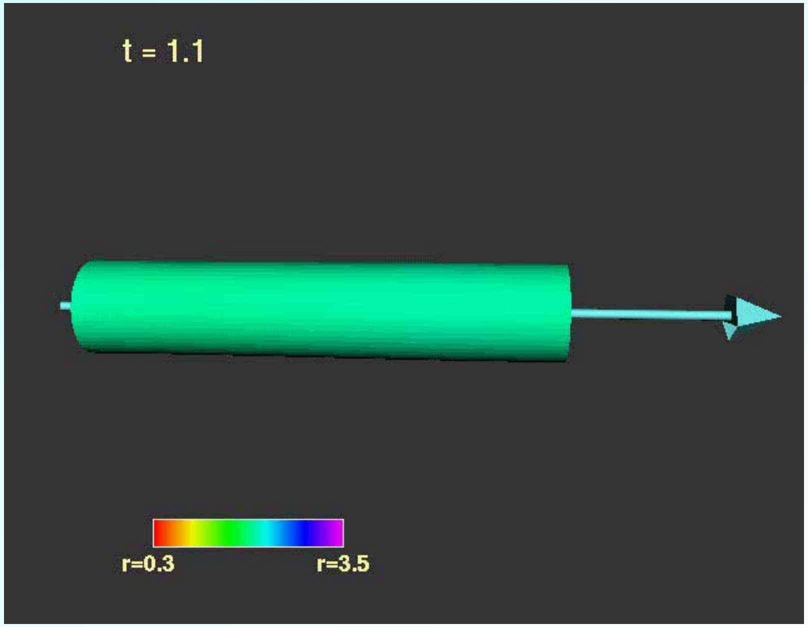
$g_{zz}(t,r,z)$

What do we know about the end state of Gregory-Laflamme-unstable black strings?

ISOMETRIC EMBEDDING DIAGRAMS OF APPARENT HORIZONS



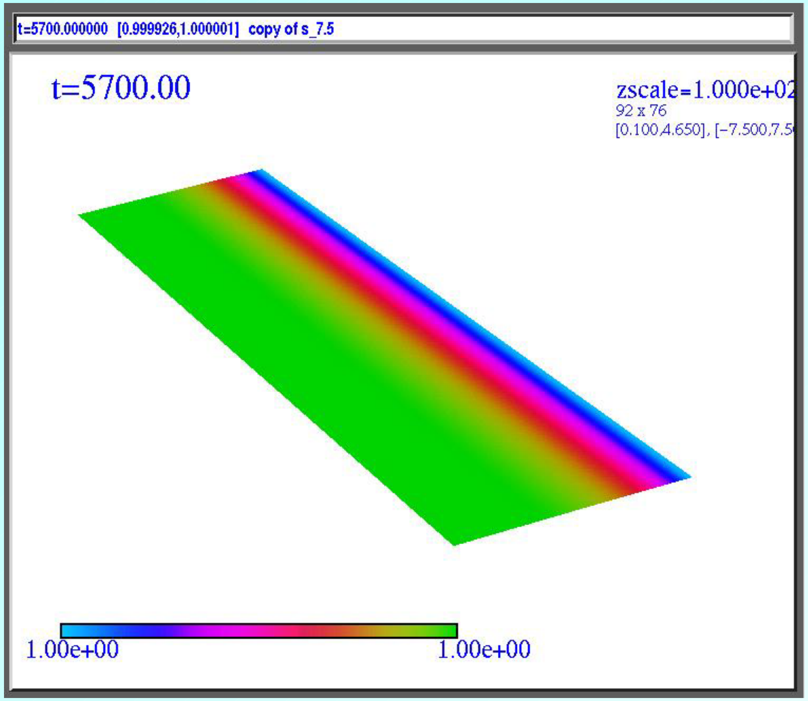
STABLE

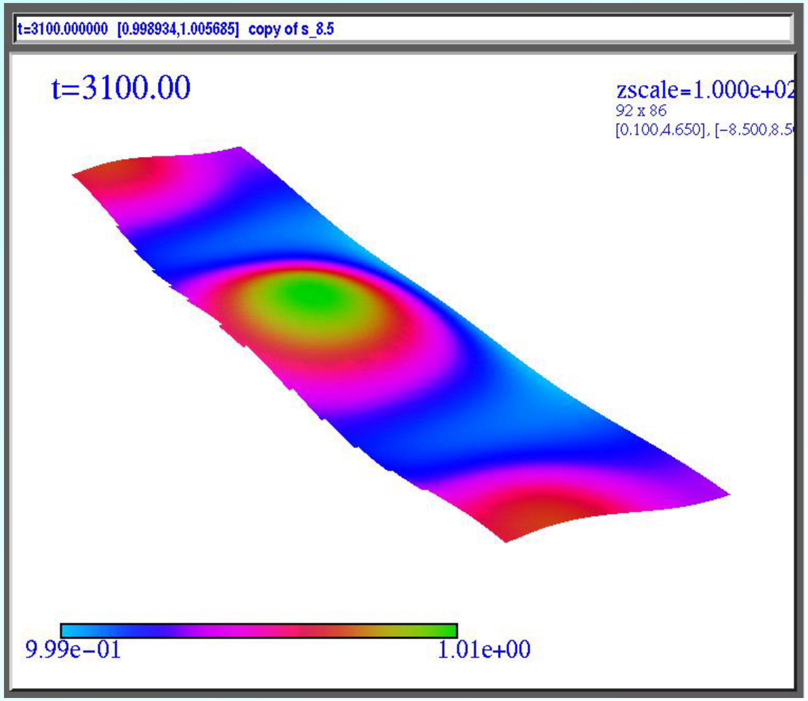


UNSTABLE

Coordinate pathology at late times apparent limiting factor

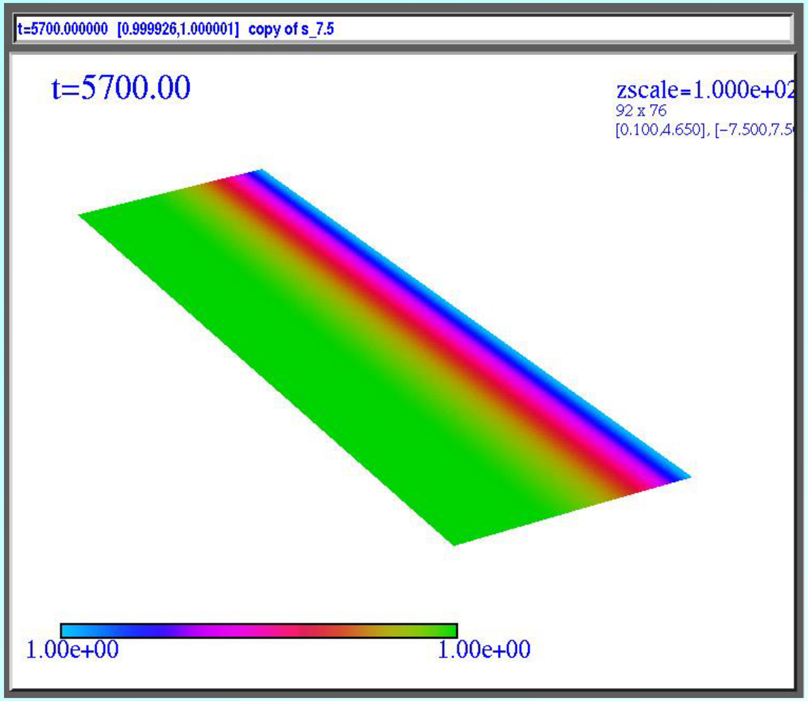
Interesting Preliminary "Results"

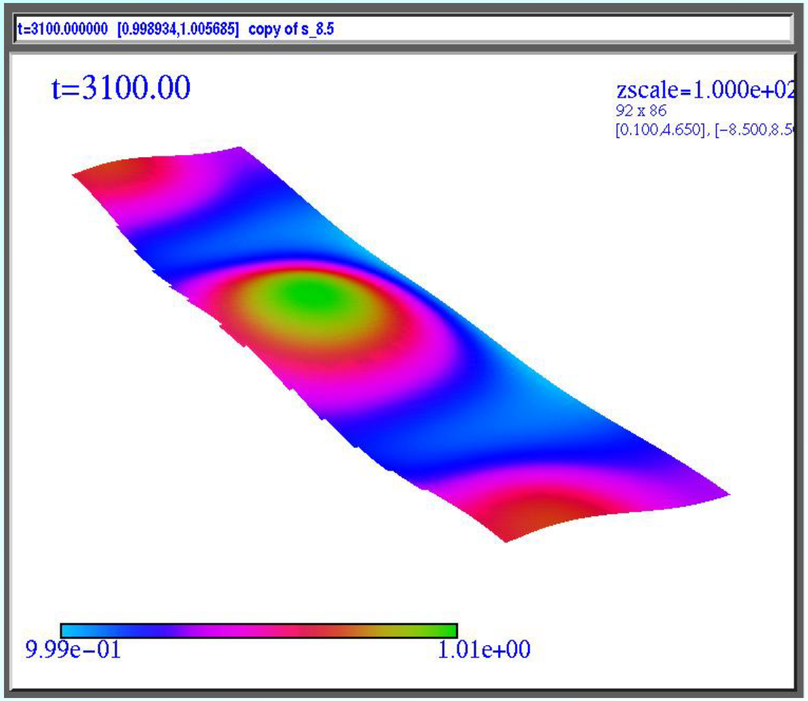




Evidence for static, non-translationally-invariant solution?

Interesting Preliminary "Results"





Evidence for static, non-translationally-invariant solution?

NO!

[Un]fortunately, "results" highly dependent on position of outer boundary, r_{MAX}

Memorable Quotes Associated with this Project

- "… so that's a nice 2+1 problem; should be straightforward to solve, right?"

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Motivation for looking for flatspace analogues of GL instability

- Numerical analysis should be MUCH easier
- Precedent for such analogues of other strong-gravity phenomena
  - Flat spacetime models exhibiting analogues of black-hole critical phenomena
    - Non linear wave maps [Liebling, Hirchsmann & Isenberg, J Math Phys, 41, 5691 (2000); Bizon & Wasserman, PRD, 62, 084301 (2000))
    - Oscillons: Klein-Gordon equation in spherical symmetry with Mexican Hat potential [Honda & Choptuik, PRD, 65, 084037, (2002)]

Motivation for looking for flatspace analogues of GL instability

```
[illegible]
```

[illegible]

[illegible]

```
end subroutine [illegible]

      [illegible]
module [illegible]
  implicit none

  integer,         save :: [illegible]
                           [illegible]
  logical,         save :: [illegible]

  integer, save :: [illegible]
  real*8, save :: [illegible]
                   [illegible], lambda, [illegible]

end module [illegible]
```

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Are there special relativistic analogues of black strings?

I don't know yet!

But I'm willing to bet there are!!

Ideas

- Skyrme type solutions
- Wave maps

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- Wave maps
- YOUR IDEA HERE