BINARY BLACK HOLE GRAND CHALLENGE UPDATE

- Formalisms

- The Initial Value Problem

- Black-Hole-Excising Techniques

- Computational Infrastructure

- Current State and Outstanding Problems

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Binary Black Hole Grand Challenge Personnel
`http://www.npac.syr.edu/projects/bh/`

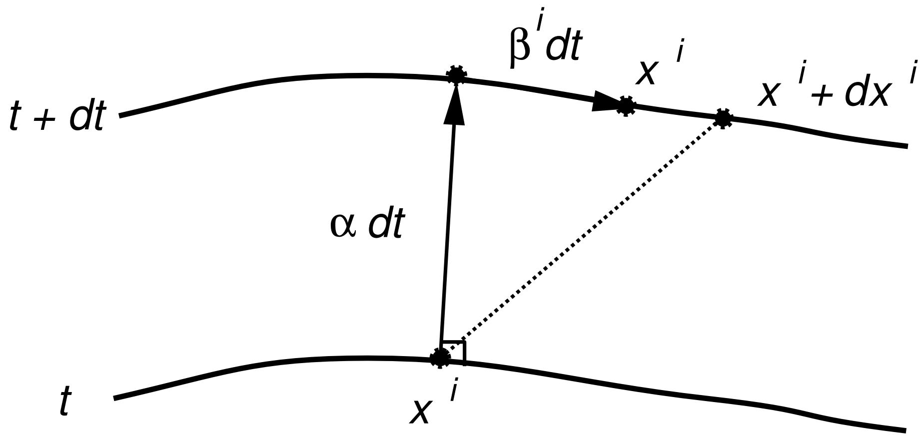
- UT Austin: MATZNER, BROWNE, CHOPTUIK, Huq, Hirschmann, Parashar; Nielsen, Liebling
- NCSA/Potsdam: SAIED, SAYLOR, SEIDEL, SHAPIRO, SMARR, Abrahams, Anninos, Baumgarte, Masso
- UNC Chapel Hill: EVANS, YORK; Rupright
- Cornell U: TEUKOLSKY, Cook, Scheel; Landry
- Syracuse U (NPAC): FOX, Haupt, Klasky
- U Pittsburgh: WINICOUR, Gomez, Marsa; Lehner
- Northwestern U: FINN, Kidder
- Penn State: LAGUNA, Papadopoulos
- Washington U: Suen (associate)
- University of South Africa: Bishop (associate)

The 3 + 1 (ADM) Formalism

- Pose general relativity as a dynamical theory (geometrodynamics).

  View geometry of spacetime as "time history" of the geometry of a spacelike hypersurface ("instant of time"), $\Sigma(t)$

- Geometry of spacetime is described by a 4-metric, ${}^{(4)}g_{\mu\nu}$
  Geometry of Σ is described by a 3-metric, g_{ij}



$$
\begin{aligned}
{}^{(4)}ds^2 &= {}^{(4)}g_{\mu\nu}dx^\mu\, dx^\nu \\
&= -\alpha^2\, dt^2 + g_{ij}\left(dx^i + \beta^i dt\right)\left(dx^j + \beta^j dt\right)
\end{aligned}
$$

- Two types of variables:

  1) Kinematical: lapse function, α and shift vector, β^i

  2) Dynamical: three metric, g_{ij}

The 3 + 1 (ADM) Formalism (cont.)

- Extrinsic curvature: K_{ij}

  Describes manner in which $\Sigma(t)$ is embedded in space-time. Can be viewed as the "velocity" of g_{ij}

$$K_{ij} = \frac{1}{2\alpha}\left(-\frac{\partial g_{ij}}{\partial t} + D_i\,\beta_j + D_j\,\beta_i\right)$$

  where D_i is the 3-covariant derivative: $D_i\, g_{ij} = 0$

- Einstein equations (vacuum):

$$G_{\mu\nu} = 0$$

  Constraint equations:

$$G_{0\nu} = 0$$

  Evolution equations:

$$\frac{\partial g_{ij}}{\partial t} = -2\alpha K_{ij} + D_i\,\beta_j + D_j\,\beta_i$$

$$\frac{\partial K_{ij}}{\partial t} = \mathcal{L}_\beta K_{ij} - D_i D_j \alpha + \alpha\left(R_{ij} - 2K_{ik}K^k{}_j + K_{ij}K\right)$$

  where R_{ij} is the 3-Ricci tensor, $K \equiv K^i{}_i$, and $\mathcal{L}_\beta$ is the Lie derivative along β^i.

Hyperbolic Formulations of Einstein's Equations

(York, Choquet-Bruhat, Abrahams, Bona, Masso)

- Standard 3+1 formulation of Einstein's equation yields set of PDEs which is generically non-hyperbolic.

  Possibility of ("gauge") information "propagating" at superluminal speeds: hampers black-hole excising techniques

- Hyperbolic formulation: Characteristic speeds are either 0 (some dynamical variables simply "Lie-dragged" along normal to hypersurfaces) or local light speed. Information only propagates along physical light cones.

  Hyperbolic formulations ("flux-conservative form") in principle allow techniques developed for fluid dynamics to be carried over to NR calculations

  Aids radiation extraction and formulation of boundary conditions (inner and outer)

- Einstein evolution equations:

$$\partial_t u + \partial_i F^i(u) = S(u)$$

  First order flux-conservative, hyperbolic form: $S(u)$ contains no spatial derivatives of dynamical variables, u.

- Possible disadvantage: lose some flexibility in choice of coordinates

The Initial Value Problem for Two Black Holes

(Ō Murchadha & York, Cook et al)

- Constraint equations

$$G_{0\nu} = 0$$

are generically a system of 4 coupled, non-linear elliptic equations for $\{g_{ij}, K_{ij}\}$; setting up initial data for a general black hole collision is non-trivial

- Key problem: Which of the $\{g_{ij}, K_{ij}\}$ should be freely specified, which should be determined via solution of the constraints?

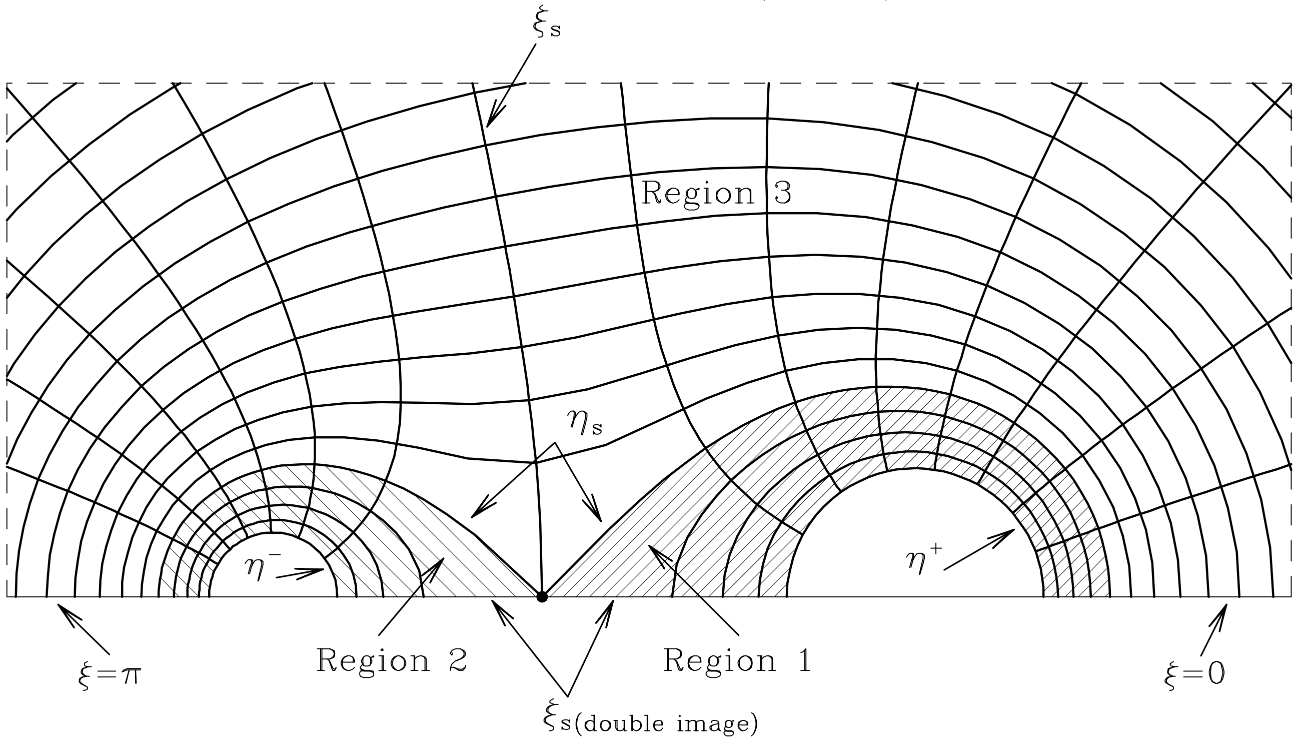
- General strategy: use conformal scaling and "spin-decomposition" of K_{ij}; constraints $\rightarrow$ system of quasi-linear elliptic equations for four potentials: $\{\psi, X^i\}$

- Momentum constraint can be solved analytically, Hamiltonian constraint requires numerical solution for ψ, typically using multi-grid technique.

$$g_{ij} = \psi^4\,\hat{g}_{ij} = \psi^4\,f_{ij}$$

The Initial Value Problem for Two Black Holes (cont.)

- State-of-the-art Hamiltonian Constraint Solver: (**Cook**). Uses special-purpose Čadež coordinates, (η, ξ, ϕ):



- Discretization is precisely $O(h^2)$, Richardson extrapolation can be applied to get $O(h^4)$ or better results. Solutions can be determined to essentially arbitrary accuracy

- Key remaining problem: How to generate realistic initial data—i.e. data for two holes with relatively small separation but representative of result of inspiral from wide separation.

Black Hole Excising Techniques

- Fundamental Problem: Black hole spacetimes contain physical singularities–apparently must be avoided at all costs in calculations

- Traditional Approach: Use coordinate freedom (choice of lapse and shift) to "freeze out" evolution in vicinity of physical singularity

- Spherically Symmetric Example: Choose coordinates r, t so that metric takes on "time-dependent-Schwarzschild" form:

$$ds^2 = -\alpha(r,t)^2\, dt^2 + \left(1 - \frac{2m(r,t)}{r}\right)^{-1} dr^2 + r^2\, d\Omega^2$$

  Consider collapse of ball of matter to form black hole: when horizon formation is imminent, then near the Schwarzschild radius $r = R_S$

$$\alpha \to 0 \qquad \left(1 - \frac{2m(r,t)}{r}\right)^{-1} \to \infty$$

  Physical singularity avoided, but coordinate singularity develops, code will crash on the order of black-hole dynamical time, $M = R_S/2$

Black Hole Excising Techniques (cont.)

- Excising Approach (**Unruh**): By definition, interior of black hole is out of causal contact with exterior spacetime. Simply remove regions inside event horizons from computational domain.

- Event horizon location not known until after computation is completed, use as inner boundary some surface known to lie within the event horizon (EH).

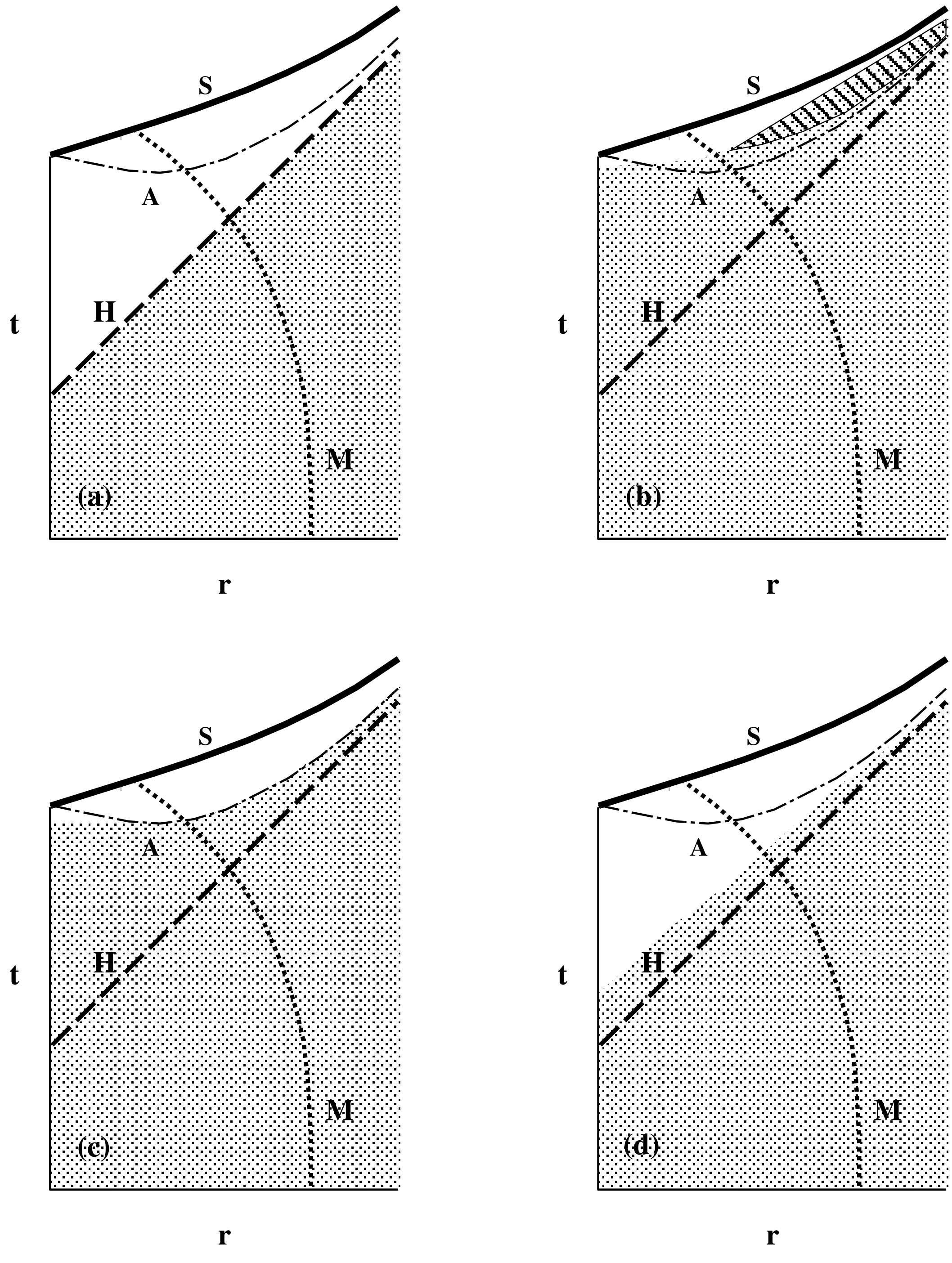
- Apparent horizon (AH): Two surface defined at some instant of time on which divergence of outgoing null rays vanishes—outgoing light rays emanating from surface "hover" at constant radius

- Assuming cosmic censorship, AH will always lie within EH, AH can be located at an instant of time by solving non-linear elliptic equation (finite-difference techniques (**Thornburg, Huq**) or spectral methods (**NCSA/Wash-U, Cornell groups**))

- Computational boundary is essentially a characteristic surface: modulo possible problems from non-hyperbolicity, don't need boundary conditions

- Efficacy of approach demonstrated in several spherically symmetric calculations (**Seidel & Suen, Scheel et al, Marsa & Choptuik, Anninos et al**); now implemented in 3D codes, some encouraging preliminary results (**Potsdam/NCSA/Wash U**)

Black Hole Excising Techniques (cont.)



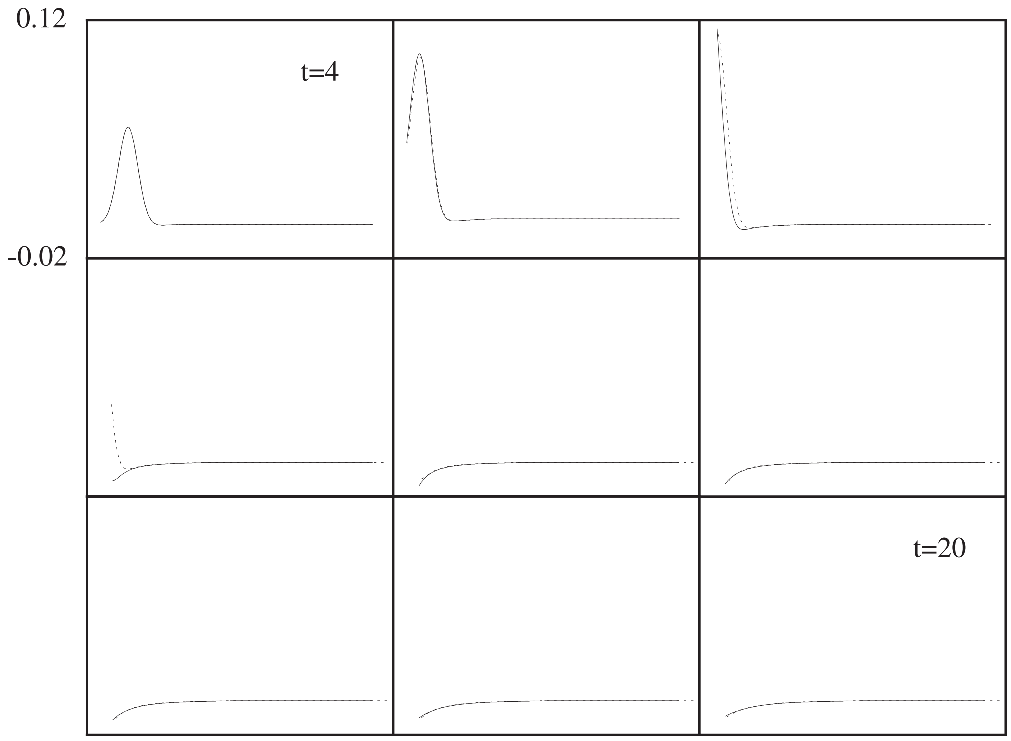
Black Hole Excising Techniques (cont.)

- Sample Calculation: Massless scalar field collapse onto a black hole (**`Marsa & Choptuik`**)

- Use Ingoing-Eddington-Finkelstein coordinates:

$$ds^2 = a^2\left((2\beta - 1)\ dt + dr\right)(dt + dr) + r^2 d\Omega^2$$

Continuously track location of outer-most apparent horizon, "throw away" grid-points inside AH.



- Plot shows time series of scalar field amplitude $\phi(r,t)$ with apparent horizon at left of each frame; evolution can be continued essentially indefinitely. Mass of scalar field is approximately equal to initial black hole mass.

Computational Infrastructure

- Part of GC charge is to "push the envelope" vis a vis large-scale computation with particular emphasis on the effective use of massively parallel architectures.

- **Observation:** Our codes have tended to be remarkably homogeneous from a "high-level" point of view: Almost all have employed low order (second-order) finite difference techniques on single mesh, and have had the following structure:

```
Read (initial) state
for NUM_STEPS
  for NUM_UPDATES & maybe until convergence
    U (Grid Function(s)) -> Grid Function(s)
  end for
end for
Write (final) state
```

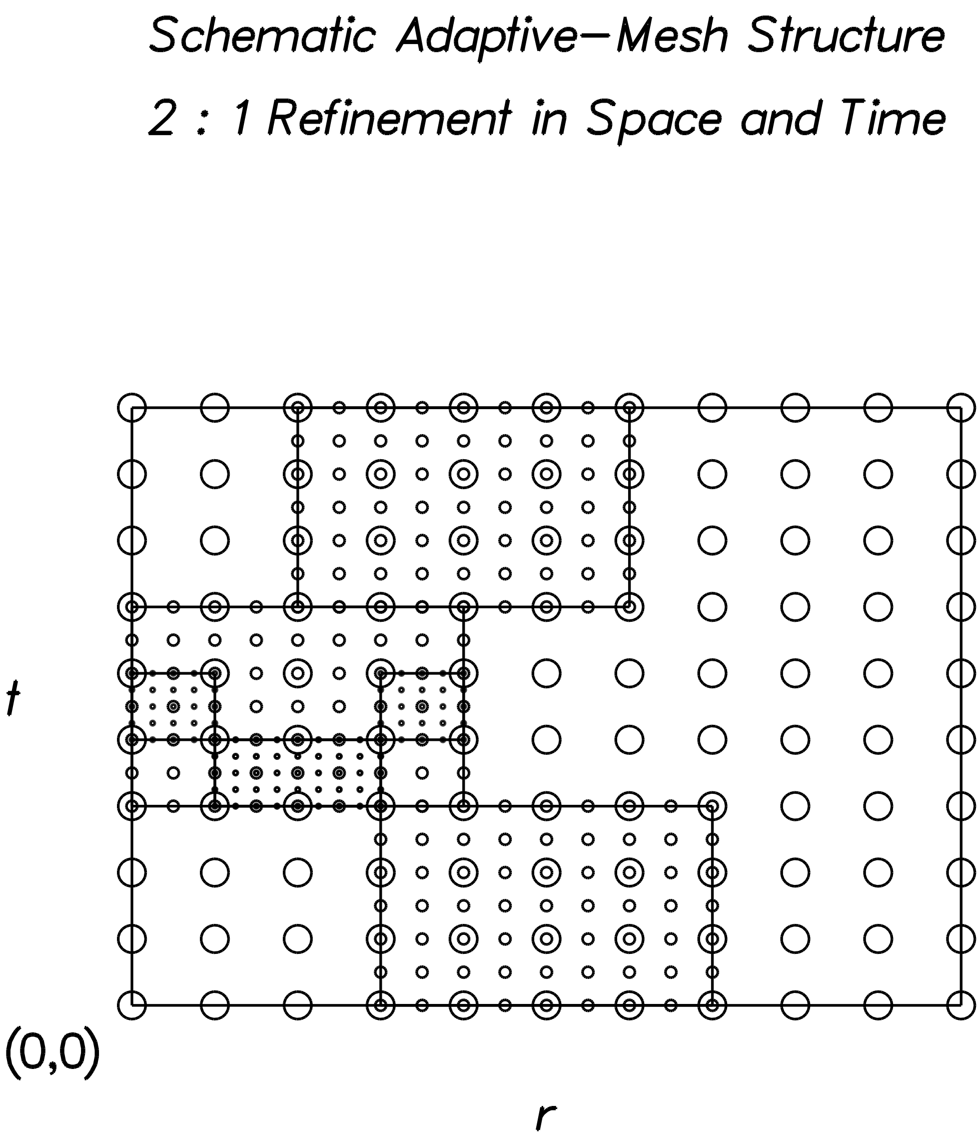
- Most of the hard work in developing a new code involves the construction of stable, accurate updates, `U`

- Also clear that significant dynamic range in black-hole problems such as binary coalescence means that adaptive-mesh-refinement (AMR) algorithms essential for efficient computation

- Ultimate goal: allow relativist to concentrate on developing stable, uni-grid code on serial architecture: parallelism and adaptivity to be "automatically" provided by the infrastructure

Computational Infrastructure: AMR

- Adopted AMR approach due to `Berger & Oliger` which achieves adaptivity via nested structure of individually-uniform grids:



- Placement of component grids controlled dynamically via local truncation error estimates

Computational Infrastructure: DAGH

`http://wwwrel.ph.utexas.edu/Members/parashar/toolkit.html`

- Computer scientists on project (**`Parashar, Browne`**) have implemented MPI-based infrastructure which provides full support for

  (1) Parallelization (distributed-memory architectures) of regular, lattice-based calculations with local communication patterns (i.e. typical finite-difference codes)

  (2) Berger & Oliger style AMR

- System is called DAGH (Distributed Adaptive Grid Hierarchy), implemented in C++ (using MPI) but designed to easily interface with update routines written in Fortran 77, Fortran 90 or C.

- Key features:

  - Automatic dynamic partitioning and load distribution

  - Shadow grid hierarchy for memory-efficient truncation error estimates

  - Maintains data locality among different refinement levels (space-filling curves)

- Currently runs on IBM SP2, Cray T3D, Networked Workstations (RS6000, SGI)

Computational Infrastructure: Status

- Evaluation and de-bugging continues, but DAGH is already integral part of BBH GC effort, and has been adopted by other groups doing CFD, reservoir simulation, etc.

- Has been used extensively in uni-grid context, particularly by Cornell group working on SP-2, AMR for IVPs (neutron stars)

- Most benchmarks (not many exist!) show very good performance; in several preliminary cases, DAGH application ran faster than hand-coded MPI application

- Full AMR code for 2D linear wave-equation implemented (**`Huq, Parashar`**), currently being rigorously tested; will serve as basis for general AMR driver.

- I/O support remains outstanding issue; collaboration with NCSA groups underway, currently all I/O done through dedicated processors, **`.hdf`** support provided, hooks for other facilities in place

Current State of Main Code Effort

- Personnel involved in main code efforts (Cornell/Texas/Syracuse) recently agreed to focus effort on ADM code; Empire code (based on Choquet-Bruhat/York hyperbolic formalism) shelved.

- ADM code: Based on traditional 3+1 formalism, currently implemented in Fortran 90, runs on pretty well anything (C90, Power Challenge, SP-2, ...)

- Code implements black-hole excising, as well as "causal differencing" designed to ensure that numerical domain of dependence includes physical domain of dependence

- Resolution limited to 128^3 (unigrid) in both cases; significantly less (65^3) for quick-turnaround runs.

- Code currently *unstable* for single Schwarzschild hole, apparently due to treatment of inner boundary

- Lots of things to try, but nature of equations, differencing, "irregular" inner-boundary makes detailed stability analysis difficult

Additional Outstanding Problems (partial list)

- Choice of coordinate system: Almost certainly most important unsolved problem:
  - Black holes need to move through computational mesh while interiors of holes are simultaneously excised
  - We have essentially no experience with formulating coordinate conditions which will allow for such propagation while remaining non-singular
  - Intrinsic angular momentum of holes will only make matters worse
  - Differencing near horizons may need to be quite intricate to ensure convergent solution
- Will excising work in highly dynamic, asymmetric situations?
- Does horizon excising require a hyperbolic formulation?
- If so, can the problem of "coordinate shocks" (**`Alcubierre`**) be avoided?

Additional Outstanding Problems (cont.)

- How can we set up near-plunge initial data which is astrophysically realistic?

- How should outer boundary be handled: will AMR techniques and standard radiation extraction methods suffice, or must we match to a characteristic evolution to get an accurate estimate of gravitational radiation?

- AMR technology crucial if we want to solve this problem in the near future: will Berger & Oliger approach suffice, or will some more flexible approach be required?

Prognosis

- PROVIDED

  - Current and future stability problems are overcome expeditiously

  - A good coordinate prescription for generic translating, rotating holes is found

  - Some form of "outgoing" radiation conditions can be stably imposed at a reasonably finite radius

  THEN odds are good that we'll be able to crudely simulate inspiral within 18 mos. Waveform accuracy???

- Valuable infrastructure *is* being developed

- Nature of calculations is such that once stable, adaptive algorithm is in place, rapid increase in available computational capacity (which we can reasonably assume) will *quickly* lead to more realistic, and accurate simulations

- Have and will learn interesting things about strong-field gravity along the way