RECENT DEVELOPMENTS IN BLACK HOLE CRITICAL PHENOMENA

- Context, Areas of Expertise & State-of-the-Art in NR
- The Threshold of BH Formation: Basic Picture
- Basic Types & Properties of Critical Solutions
- Coordinate Choices & Black Hole Excision
- SU(2) Yang-Mills Collapse
  - Static Solutions: "Bartnikons" and colored black holes
  - Threshold Critical Behaviour (magnetic ansatz)
  - Forming Colored Black Holes via Fine Tuning
  - Instability of Magnetic Critical Solutions

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Context

- Facts of NR/BH simulation relative to Newtonian-PN gravity

  - Field approach essential: No viable/competitive particle formulation of vacuum Einstein equations. e.g. General relativistic SPH is SPH on fixed background. PM methods *not* widely-used, excepting **Shapiro & Teukolsky**.

  - Computation *must* be done with realistic boundary conditions (i.e. no PBCs). Asymptotic flatness is arguably good enough for foreseeable future.

  - BH singularities are a *real concern* and must be dealt with effectively. For "astrophysical" (BH exterior) purposes, BH excision provides basic approach/mechanism.

  - Dynamical black hole problems will *always* require *significant* dynamic range—adaptivity will be crucial for efficient computation.

Context (cont.)

- Facts of NR/BH simulation relative to Newtonian-PN gravity
  - Finite element and/or spectral methods and/or "high-order" methods have *not* found general applicability to dynamic, vacuum Einstein equations—second-order finite-differencing on logically-Cartesian meshes is the *only* widespread approach.
  - *Stability problems have been, are, and will continue to be SEVERE*. (Where's the cyclist when you pass her on a hilly road?)
  - Not much intuition to guide us—simulation potentially *very* powerful, but generally must be done more fastidiously.
  - Manpower limited. Many of the best young researchers currently leave the field shortly after the PhD (**Guenther, Klasky, Marsa, Walker, Brandt,** ...)

Areas of Expertise/Deliverables

- Expertise in solving time-dependent (hyperbolic, parabolic) non-linear PDEs using multi-scale finite difference techniques

- Expertise in solving time-independent (elliptic) non-linear PDEs using multi-scale finite difference techniques

- High level prototyping languages which enable those with expertise to more rapidly solve problems, and those without expertise to more rapidly gain it (**`RNPL`**, **`Hirschmann`**)

- Expertise in visualization and the development of interactive analysis interfaces
  - **`ser:`** Graphical analysis of 1D time-dependent PDEs
  - Although custom-made for 1D, is now widely used by those working in 3D, largely because appropriate 2- and 3D tools don't exist yet
  - Could change on a short time scale, but will provide interesting/useful graduate work in any case

Areas of Expertise/Deliverables (cont.)

- Expertise in parallelization of block-structured finite-difference applications
  - Parameter-space parallelism (embarrassingly parallel) (automatic in `RNPL`)
  - `DAGH` (BBH GC product: MPI-based support for Berger & Oliger-style AMR)
  - `PUGH` (MPI Potsdam/Cactus Collaboration product: MPI-based support for unigrid codes)
- Training/guiding of students in essentially arbitrary sub-fields of computational physics

State-of-the-Art in NR

- Parameter-space studies *always* add "dimensionality" to a problem.

- 1D (spherical)
  - Interactively computable on single workstation, multiple processors *very* useful for parameter space surveys
  - Still lots of room for improvement, extensions
  - Certainly not *solved* problem: In addition to this work, see e.g. **`Neilsen and Choptuik gr-qc/9812053`** for some rather basic, but previously uncomputed, calculations of self-gravitating, perfect-fluid flows—also observe severe confusion associated with spherically-symmetric self-similar Einstein/fluid solutions.
  - Good training ground for students
  - Good source of calibrating solutions for 2D, 3D

State-of-the-Art in NR (cont.)

- 2D (axisymmetric)
  - Interactively computable on medium-sized machine (32 procs?), but largely abandoned due to seductivity of 3D and problems on axis
  - Almost certainly much new physics to be learned via careful, detailed study in 2D
  - Black hole excision not yet implemented/demonstrated
  - AMR not yet implemented/demonstrated
  - Main focus of my current research—no significant results yet for strong field calculations.

State-of-the-Art in NR (cont.)

- Typical reaction to poor-man's parallelism (simultaneous interactively monitored computations)

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From:  root@crunch.mrl.ucsb.edu (Super-User)
Message-Id:  <199905150221.TAA13145@crunch.mrl.ucsb.edu>
To:  choptuik@crunch.mrl.ucsb.edu
Subject:  ONLY RUN IN THE NQS QUEUES
Status:  R


crunch% qstat -a

Request I.D. Owner Queue

coarsen600K 77360 thomas long-8cpu
Darth_Monica 77365 helgi long-8cpu

myqsub 77358 shannon long-8cpu
Stop_running_outside_the_f...ing_queues,_choptuik

jobPdmrg 77351 sdaul long-4cpu
STDIN 77325 kalju long-4cpu
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STDIN 77364 kalju long-4cpu
STDIN 77370 kalju long-4cpu
n4b1214 77348 chris long-12cpu
myqsub 77353 shannon long-12cpu
sampleBZ 77363 nblake long-12cpu
n4b1214 77349 chris long-12cpu
```

State-of-the-Art in NR (cont.)

- 3D (no symmetries)
  - Focus of most current collaborative NR research: Potsdam, Wash U, Penn State, U Texas, NCSA, Cornell, Pittsburgh, Kyoto
  - Representative Calculation 1:
    gr-qc/9904013, Alcubierre et al
    $128 \times 128 \times 128$ with octant-symmetry
    (120 hours on 16 processor 02000, highly parallelized)
    Collapse of Brill waves to form black hole
  - Representative Calculation 2:
    gr-gc/9711078, Cook et al
    $33 \times 33 \times 65$
    (many hours on 8-head SGI, not very parallelized)
    Movement of BH through 3D mesh using excision
  - No clue (still) how to choose coordinates for realistic problems, no stable horizon excising formulation for generic case
  - No AMR
  - Underdeveloped I-O/visualization/analysis
  - Resources readily available to single group probably two orders of magnitude too small for interactive solution ($\sim$ 1 hour) to few % level, even with "fully optimized" solution (i.e. with AMR)

Summary of Students 1994—1999

- Scott Klasky (94, Matzner)
  1, 2 and 3D elliptic, parallel, AMR

- Reid Guenther (95, Matzner)
  2 and 3D time-dependent Schrödinger/Poisson, parallel, AMR

- Robert Marsa (95, Choptuik)
  1D GR/scalar, horizon excising, 2 and 3D scalar field on Kerr background, parallel

- Steve Liebling (98, Choptuik)
  1D GR/scalar, 2D scalar/EM, 2 and 3D O(3) textures, AMR

- Dae-Il Choi (98, Choptuik)
  1, 2 and 3D time-dependent Schrödinger, 3D GR/scalar, AMR, parallel

- David Neilsen (99, Choptuik)
  1 and 2D GR + fluid, AMR, parallel

- In progress at UT:
  Hawley, Honda, Ventrella, Noble

- New at UBC:
  Lai, Olabarrieta (transfer from UT), Petryk, Pretorius

SUMMARY: The Threshold of Black Hole Formation
Critical Phenomena in Gravitational Collapse

- **Rules of the BH Threshold Game**

  - Choose a model of gravitational collapse, and (generally), consider *ultrarelativistic* (*KE* dominated) initial conditions describing implosion of matter-energy

  - Parametrize initial conditions (parameter p), so that for $p < p^\star$, matter-energy implodes then completely disperses, and for $p > p^\star$, a BH is formed

  - Choose *generic* initial data, an initial bracket $[p_{\text{lo}}, p_{\text{hi}}]$, then tune to the critical parameter value, $p^\star$ via a series of dynamical evolutions

SUMMARY: The Threshold of Black Hole Formation
Critical Phenomena in Gravitational Collapse

- For given matter model, couplings, symmetry, $\cdots$ find one or more isolated critical solutions, $g^\star$

  - Threshold solutions are unstable by construction

  - Threshold solutions are *minimally* unstable—in perturbation theory, generally have only *one* unstable mode.

  - Lyapounov exponent (growth factor), λ_1 associated with critical solution can be computed "experimentally" by measurement of scaling laws, just as in statistical mechanical critical phenomena

- Particularly interesting quantity: Black hole mass at criticality

  - **Type II**: Infinitesimal, $g^\star$ contains a naked singularity

  - **Type I**: Finite

The Threshold of Black Hole Formation

- Will focus on spherically-symmetric collapse—basic picture expected to extend to *generic* collapse, but needs detailed study

- Adopt some matter model (e.g. massless scalar field) and couple to gravitational field

- Consider parametrized families of collapse solutions—i.e. vary initial conditions and see what happens

- Demand that family "interpolate" between "flat" spacetimes and BH spacetimes (as $t \to \infty$)

  - $p < p^\star$ No BH forms (complete dispersal)

  - $p > p^\star$ BH forms

The Threshold of Black Hole Formation (cont.)

- Interpolating families characterized by *COMPETITION*

- Where can energy in system end ups?

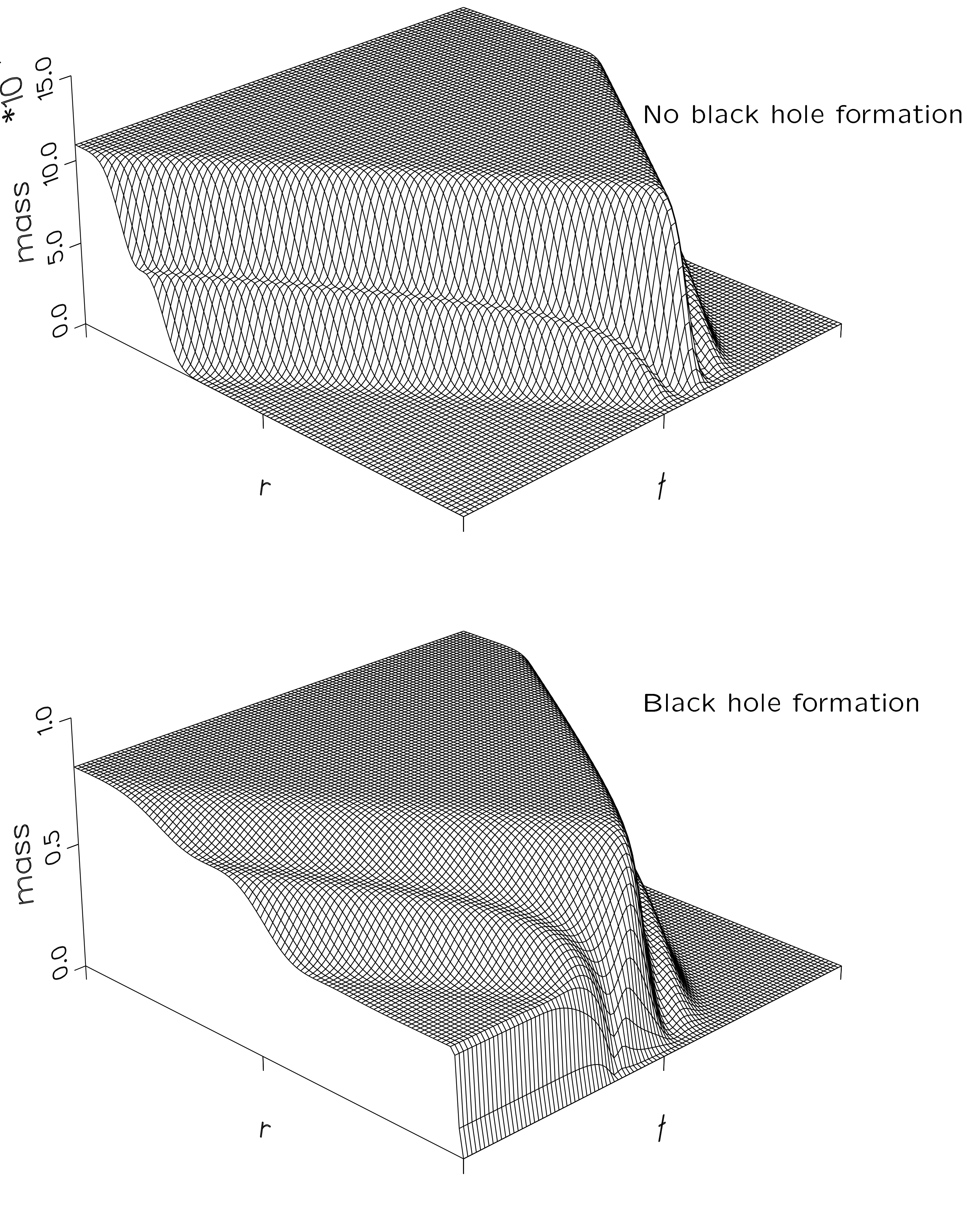
  - *K.E.* $\Longrightarrow$ $r = \infty$

  - *GRAVITATIONAL P. E.:* $\Longrightarrow$ $r < r_{BH}$

- BH formation "turns on" at threshold, or critical, parameter value, $p^\star$

Phenomena in near-threshold regime $\equiv$ *Critical Phenomena*

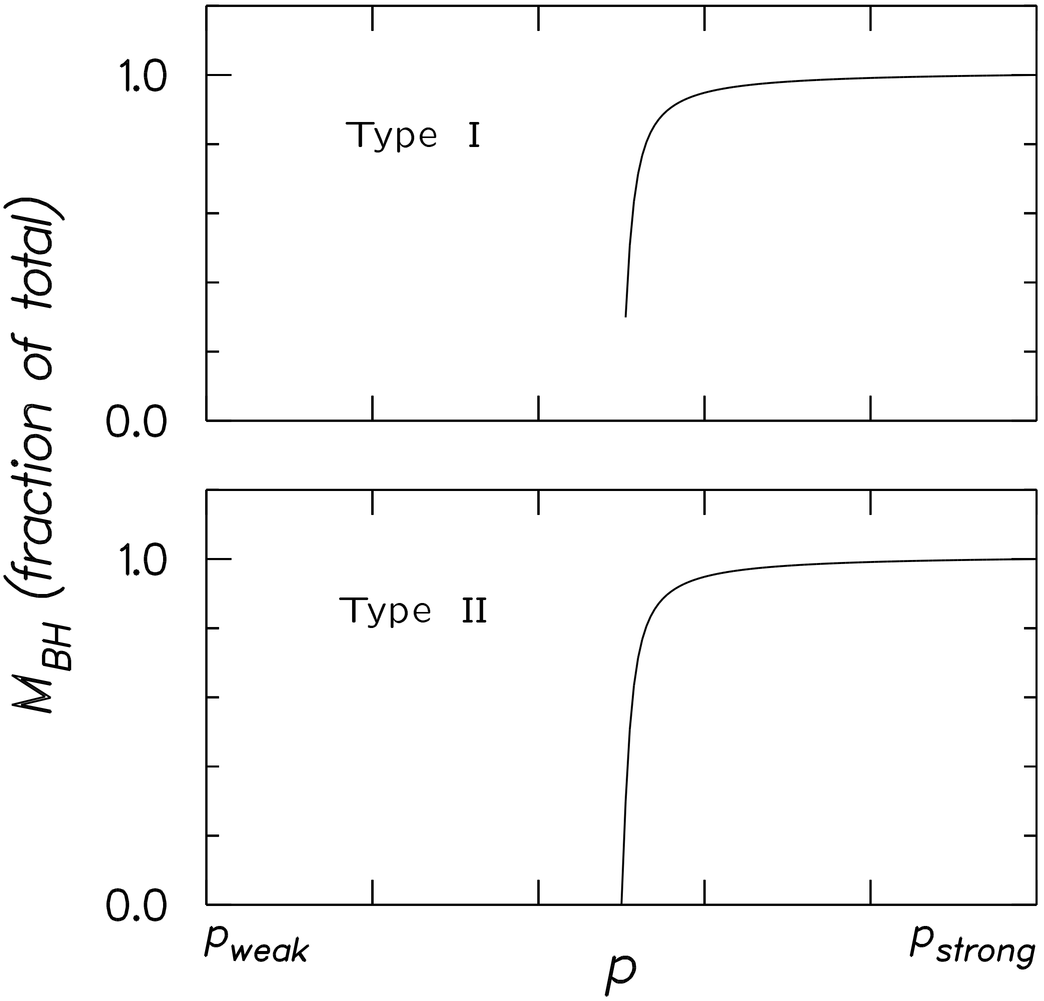
Massless Scalar Collapse: “Interpolating” Solutions



Key Question

(D. Christodoulou, 1987)

What is the behaviour of the black hole mass at a critical point for a generic interpolating family?



Critical Solutions

- Given collapse model (matter content, couplings, symmetry), find one or more isolated critical solutions, $g^\star$, of coupled Einstein/matter equations

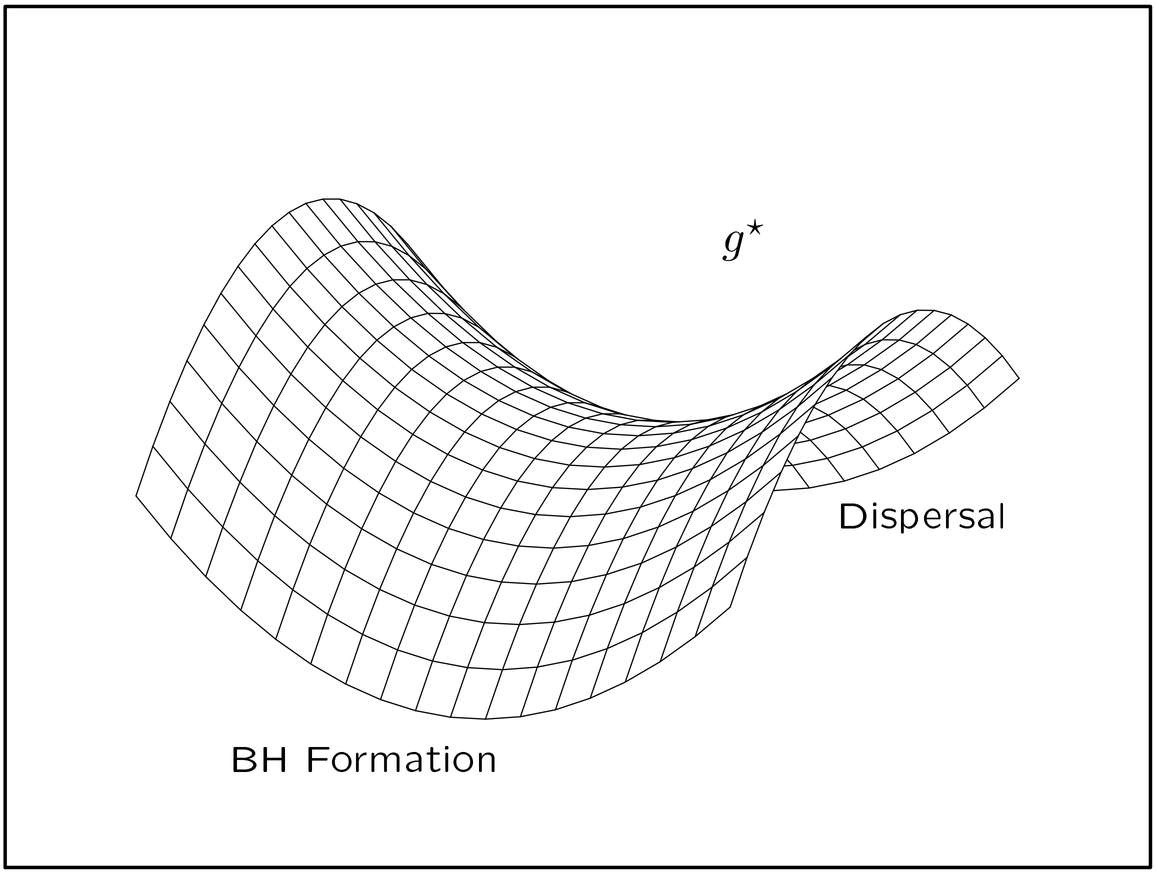
- Critical solutions are universal in the sense that they don't depend on the details of initial data $\Longrightarrow$ can be computed directly via appropriate *ansatz*

- Critical solutions are *UNSTABLE* by construction—lie on boundary between two *distinct*, stable end-states

- Consider perturbation theory about $g^\star$: Typically find precisely *one* unstable eigenmode—associated eigenvalue λ_1 has Re $\lambda_1 > 0$

(Evans & Coleman, Koike et al, Maison, Eardley & Hirschmann, Gundlach)

Critical Solutions (cont.)



- Critical solutions are *intermediate attractors* (in language of **Barenblatt**)

- Tuning $p \to p^\star \quad \equiv \quad$ tuning away single unstable mode

- Solution increasingly well described by ($\zeta \equiv \log(R/T)$)

$$g(\zeta,\tau) \sim g^\star + C_1 \exp(\lambda_1 \tau)\ f_1(\zeta)$$

(considering CSS for simplicity)

SUMMARY: Basic Types & Properties of Critical Solutions

- **Type I**—Finite BH Mass at Threshold
  - Static or Periodic
  - Reciprocal Lyapounov exponent of single unstable mode can be computed (e.g.) by measurement of lifetime

$$\tau \sim -\sigma_m \ln|p - p^\star| \qquad \sigma_m = \lambda_1^{-1}$$

  - Observed in collapse of $\boxed{\text{YM field}}$ (static), massive real scalar field (periodic), complex scalar field (both), presumably in perfect fluid

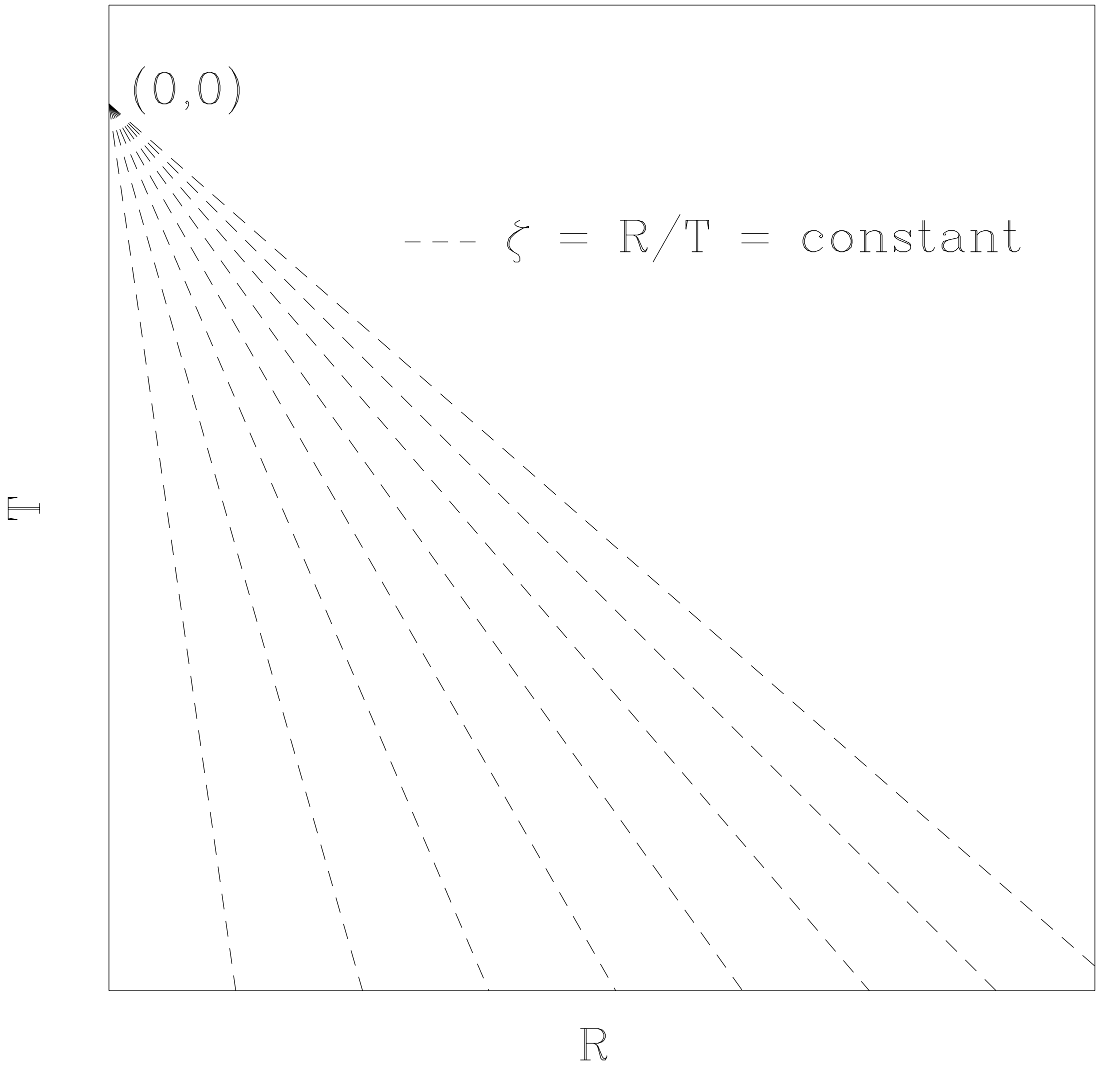
SUMMARY: Basic Types & Properties of Critical Solutions

- **Type II**—Infinitesimal BH Mass at Threshold
  - Continuously Self-similar or Discretely Self-similar
  - Reciprocal Lyapounov exponent of single unstable mode can be computed (e.g.) by measurement of mass scaling exponent

$$\ln M_{BH} \approx \gamma_m \ln|p - p^\star| \qquad \gamma_m = \lambda_1^{-1}$$

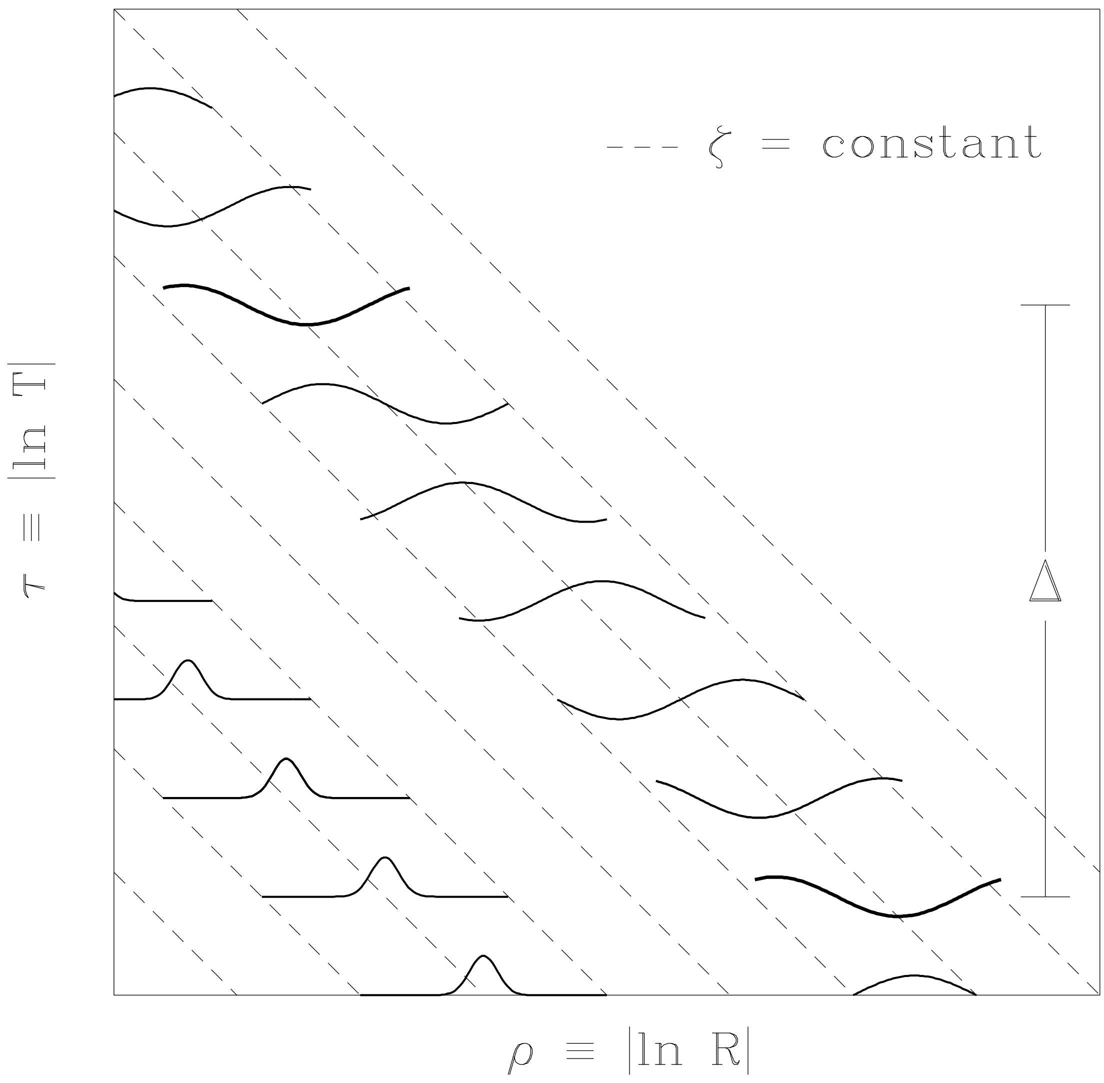
  - Observed in collapse of scalar field, gravitational field, perfect fluid, $\boxed{\text{YM field}}$, $\cdots$
  - Suggests generic mechanism to form arbitrarily small BHs and arbitrarily large & visible curvatures via tuning of initial conditions

Self-Similarity: Continuous and Discrete



- $T \equiv$ central proper time (measured from "accumulation event")
- $R \equiv$ physical (e.g. areal) radius
- Similarity variable: $\zeta \equiv R/T$
- (Naked) singularity at $(0, 0)$

Self-Similarity: Continuous and Discrete



- Continuously self-similar (CSS):

$$g^\star(\zeta,\tau) = g^\star(\zeta,\tau') \qquad \tau,\ \tau' \quad \text{arbitrary}$$

- Discretely self-similar (DSS):

$$g^\star(\zeta,\tau) = g^\star(\zeta,\tau+n\Delta) \qquad n = 0, \pm 1, \pm 2, \cdots$$

Δ—*model-dependent "echoing exponent"*

Critical Solutions—Extended Definition

One-mode unstable solution $\equiv$ *critical solution*

$g^\star$ does *not* necessarily sit at a No-BH—BH threshold

- In particular, will see example where $g^\star$ sits on the threshold between *two distinct black hole states* ($g^\star \equiv$ “hairy BH”)

Critical Phenomena—Type II Generic Features

- Arbitrarily small black holes possible (definition)
  - Sufficient fine-tuning of initial data can result in regions of spacetime with arbitrarily high curvature
  - Precisely critical solutions contain naked singularities

- Power-law scaling of black hole mass (and other dimension-full quantities)

$$M_{BH} \sim c_f \, |p - p^\star|^{\gamma_m}$$

  where $\gamma_m = (\text{Re } \lambda_1)^{-1}$

- Scaling behaviour of critical solution
  - Discrete self-similarity (scale periodicity / "echoing")
    scalar, gravitational, Yang-Mills waves
  - Continuous self-similarity
    perfect fluid, mutiple-scalar systems

Critical Phenomena—Type I Generic Features

- Smallest possible black hole has finite mass (definition)

- Critical solution is *static* or *periodic*

- Scaling law for *lifetime*, τ, of near-critical solution

$$\tau \sim -\sigma_m \ln|p - p^\star|$$

  where $\sigma_m = \left(\mathrm{Re}\ \lambda_1\right)^{-1}$

- Observed in collapse of SU(2) YM field and massive scalar field(s), and expected for “non-ultra-relativistic” fluid collapse

SUMMARY: Coordinate Choices & Black Hole Excision

- **Fact 1:** BHs contain singularities

- **Fact 2:** There are no generic algorithms for singularities

- **Implication:** BH simulations must avoid singularites or crash on a dynamical time scale $T_D \sim M_{BH}$

  - BH simulations *can* (consistently) avoid singularities

SUMMARY: Coordinate Choices & Black Hole Excision

- **Traditional Approach:** `(DeWitt et al)`
  - Use coordinate freedom (usually time slicing), to "freeze out" evolution in vicinity of physical singularity
  - Coordinate singularities appear *inevitable*. Simulations still crash on a dynamical time scale

- **Excising Approach:** `(Unruh, Thornburg, Seidel/Suen, here)`
  - No need to evolve interior of BHs—so simply excise from domain
  - Track apparent horizon or interior thereof to ensure excised region remains strictly inside BHs
  - Boundary of excised region is super-characteristic: *no* special boundary conditions required, EOM apply up to and including boundary of computational domain

- Not all coordinate choices are compatible with excising
  - Formulating appropriate choices challenging task in 3D.
  - $\boxed{\text{Good choices exist for 1D.}}$
  - 2D unstudied.

Time-Dependent, Spherically-Symmetric Gravitational Fields

- Most general form of metric in (t, r, θ, φ) coords. ($c = 1$):

$$ds^2 = \left(-\alpha^2 + a^2\beta^2\right) dt^2 + 2a^2\beta\, dtdr + a^2\, dr^2 + r^2 b^2\, d\Omega^2$$

$$\alpha \equiv \alpha(r,t) \quad \beta \equiv \beta(r,t) \quad a \equiv a(r,t) \quad b \equiv b(r,t)$$

- **Note:** $R \equiv rb$ measures proper surface area.

- (Gravitating) mass well defined in spherical symmetry (at least in vacuum regions)

$$m(R,t) = \frac{1}{2} R \left(1 - R^{;\mu} R_{;\mu}\right)$$

m, dm/dR useful diagnostic quantities

Time-Dependent, Spherically-Symmetric Gravitational Fields

- Extrinsic Curvature: Variables dynamically conjugate to a, b:

$$K^i{}_j \equiv \mathrm{diag}\left(K^r{}_r(r,t), K^\theta{}_\theta(r,t), K^\theta{}_\theta\right)$$

$$\boxed{\textit{Choice of coordinate system} \equiv \textit{Specification of } \alpha \textit{ and } \beta}$$

- α (often, and here) fixed via condition on mean extrinsic curvature, K

$$K \equiv K^i{}_i = K^r{}_r + 2K^\theta{}_\theta$$

  - $K(r,t) = 0$ maximal slicing
  - $K(r,t) = K^r{}_r(r,t)$ polar slicing

- β (often, and here) fixed via condition on 3-metric components:

  - $b(r,t) = 1$ areal coordinates

Treatment of BH Singularities: Traditional Technique

- Consider collapse of regular (singularity–free) initial data

- If a black hole forms, expect *physical* singularity in interior

- Physical singularities evidently (?) must be avoided in numerical solution of Einstein equations

- Traditional approach uses coordinate freedom (usually slicing freedom) to “freeze out” evolution in vicinity of physical singularity

- *Maximal slicing* ($K = 0$) canonical example: slices penetrate event horizon, but “pile up” without intersecting physical singularity

- *Physical singularity avoidance* comes at a price: if slices (significantly) penetrate event horizon, *coordinate* singularities essentially inevitable, just as pathological for numerical solution

Treatment of BH Singularities: Black Hole Excision
(Unruh (1982), Seidel & Suen (1991))

- By definition, interior of black hole can not influence exterior

- If interest is only in external spacetime (outside BHs), then can, in principle, "throw away" black hole interior

- Event horizon (EH) is *global* quantity—cannot be located without complete knowledge of spacetime

- Approximate location of EH at any time by finding the *apparent horizon* (AH)—outermost marginally trapped surface)—modulo cosmic censorship $r_{EH} \geq r_{AH}$

- **Strategy:** Continuously check for AH formation: once detected, track position, $r_{AH}(t)$ and excise $r < r_{AH}(t)$ from solution domain

- Location of AH moves outward *at least* as fast as speed of light, i.e. AH locus is a characteristic surface

- Do not have to specify boundary conditions at $r_{AH}(t)$, simply use evolution equations (appropriately one-sided) on AH as well as in exterior region

Specific Coordinate Systems Used

I) Polar/Areal Coordinates

- **slicing condition:** $K = K^r{}_r$ (polar slicing)
- **shift condition:** $b = 1$ (areal coordinates)

$$ds^2 = -\alpha^2 dt^2 + a^2 dr^2 + r^2 d\Omega^2$$

- Natural generalization of Schwarzschild coordinates: *cannot* penetrate apparent horizons, thus cannot use with excision.
- Minimizes algebraic complexity of Einstein equations, easy to design stable finite-differencing schemes

II) Maximal/Areal Coordinates

- **slicing condition:** $K = 0$ (maximal slicing)
- **shift condition:** $b = 1$ (areal coordinates)

$$ds^2 = (-\alpha^2 + a^2\beta^2)dt^2 + 2a^2\beta\, dtdr + a^2 dr^2 + r^2 d\Omega^2$$

- Coordinates *can* penetrate apparent horizons, can use with excision.

SUMMARY: SU(2) Yang-Mills Collapse

- Esoteric, (but) widely studied, model of gravitational collapse. Couples non-Abelian gauge field to GR. Theory has both magnetic and electric charge density.
  - Model A is spherically symmetric and purely magnetic.
  - Model B, which generalizes Model A is SS and electromagnetic.
- Model A has stationary, but unstable, centrally *regular* states which arise through interaction of YM field and gravity: "lowest lying" unstable state, $W_1(r)$, has single unstable mode.
- W_1 can be generated via the BH threshold game
  - First explicit demonstration of a Type I transition

SUMMARY: SU(2) Yang-Mills Collapse

- Model A has stationary, but unstable, centrally *irregular* states which arise through interaction of YM field and gravity, "lowest lying" unstable state, $W_1^H(r)$ has *single* unstable mode. $W_1^H(r)$ describes a black hole geometry "dressed" with a non-trivial YM field in the exterior.

- $\boxed{W_1^H \text{ can be generated via the BH threshold game}}$

  - Dynamical black hole excision very successful

  - New type of critical transition, analogous to Type I

- Model A also has a Type II critical solution and coexistence of the Type I and Type II solutions

- Perturbatively, W_1 and W_1^H have an additional unstable mode in the context of model B.

- $\boxed{\text{Verified dynamically, "phase space" being studied}}$

SU(2) Yang-Mills Collapse

(``the cold sterile world of the quantum gravitationalist'')

(R. Blandford, paraphrased)

- Action for Einstein-Yang-Mills (EYM) theory:

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{16\pi G} - \frac{1}{g^2}F^a_{\mu\nu}F^{a\mu\nu}\right]$$

- Einstein equations:

$$\begin{aligned}\frac{1}{16\pi G}G_{\mu\nu} =& T_{\mu\nu}\\ =& \frac{1}{g^2}(2F^a_{\mu\lambda}F^a{}_\nu{}^\lambda - \frac{1}{2}g_{\mu\nu}F^a_{\alpha\beta}F^{a\alpha\beta})\end{aligned}$$

- Yang-Mills field equations:

$$D_\mu F^{a\mu\nu} = 0$$

SU(2) Yang-Mills in Spherical Symmetry

- Two kinds of charge density: electric & magnetic

- Purely-magnetic + "hedge-hog" *ansatz* self-consistent with spherical symmetry (static and dynamic)
  - Yang-Mills dynamics completely described by single function:

$$w(r,t)$$

    (specific component of YM connection)

- More general: Electro-magnetic + "hedge-hog" *ansatz*
  - Yang-Mills dynamics completely described by two functions

$$w(r,t), \qquad \tilde{w}(r,t)$$

    (plus two gauge functions, $u(r,t), v(r,t)$)

- New computations
  - Dynamical formation of colored black holes in the purely-magnetic model (uses excision, achieves efficient BH evolution)
  - (In)stability of purely-magnetic critical solutions in electro-magnetic model.

Static Solutions: Regular Configurations

(Bartnik and Mckinnon (1988))

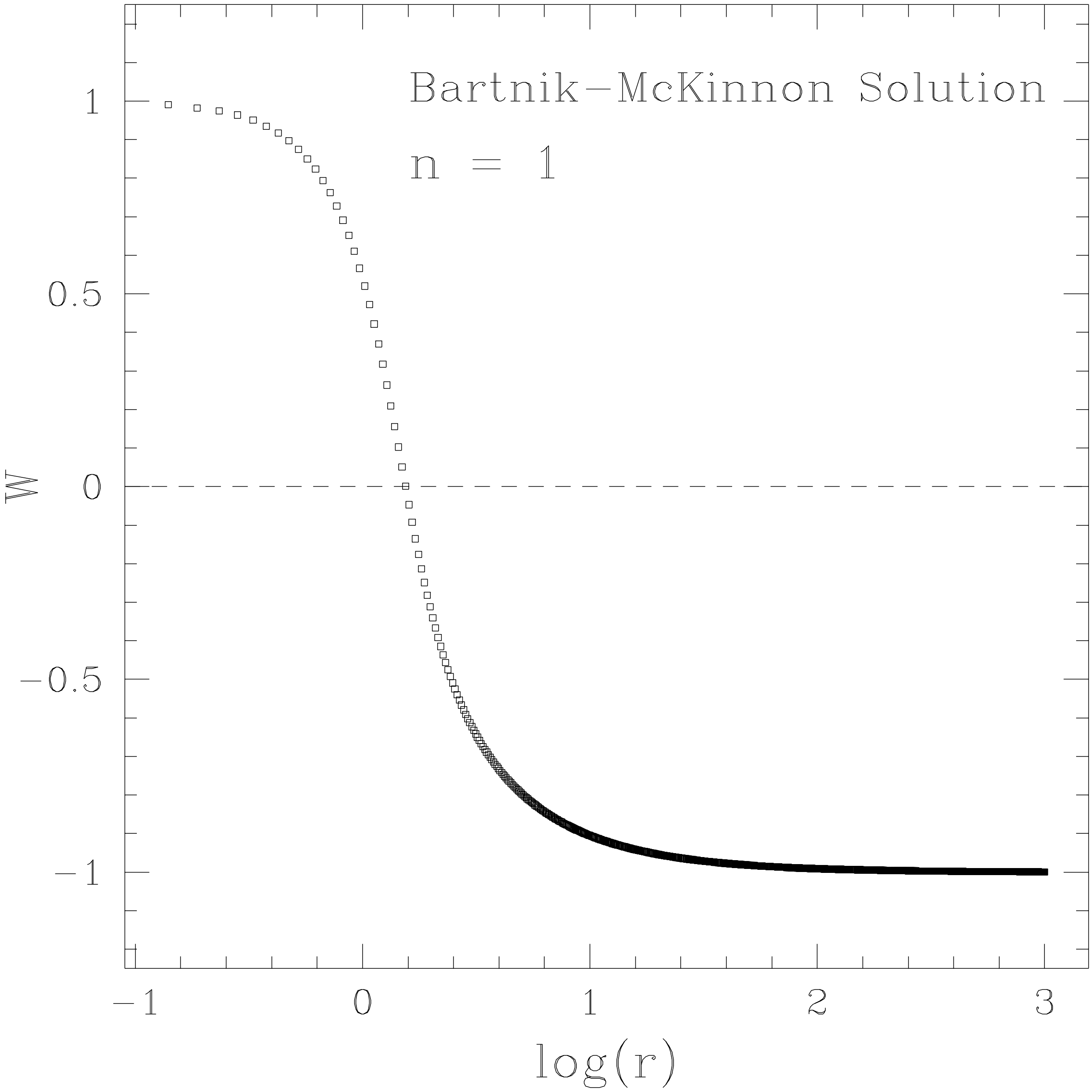
- Looked for static solutions in purely-magnetic model

- Found countable infinity of regular (at $r = 0$) solutions which can be labeled by the number, n, of zero-crossings of $w(r)$

- Somewhat surprising result since neither component theory (Einstein, Yang-Mills) admits regular static solutions: heuristically, non-linearity of YM field provides effective self-repulsion to "balance" gravity; non-trivial vacuum structure plays important role

- Was soon shown that all solutions, $w_n(r)$, are unstable with precisely n unstable modes in the context of the magnetic ansatz, and $2n$ unstable modes in the context of the more general description

$n = 1$ **Bartnik-Mckinnon Solution**



Static Solutions: Colored Black Holes

(Bizon (1990); Volkov and Gal'tsov (1990))

- Same assumptions which lead to Bartnik/Mckinnon solutions, but this time spacetime has a horizon rather than regular center

- Again find countably infinite families of solutions (again labeled by number of zero-crossings of $w(r)$), but now there is a *continuum* of such families, rather than just one

- Each family characterized by mass of black hole (radius of horizon) which effectively becomes a freely specifiable parameter

- Solutions violate the letter (if not the spirit) of the no-hair theorems—are black holes dressed with non-trivial YM configurations (YM "hair") a.k.a. *colored black holes*

- Also shown to be unstable with n or $2n$ unstable modes, depending on whether one works in the purely magnetic, or general framework

Threshold Critical Behaviour (magnetic ansatz)
(Choptuik, Bizon and Chmaj (1996))

- Studied dynamics of collapsing YM field, in polar/areal coordinates; found two distinct types of black-hole threshold behaviour:
  - **Type I behaviour**: As $p \to p^\star$, dynamics asymptotes to $n = 1$ Bartnik-Mckinnon solution, with "lifetime" of configuration given by

    $$\tau \approx -\sigma \ln|p - p^\star|$$

    where $\sigma \approx 0.552$ is the reciprocal Lyapunov exponent of the single unstable mode for the $n = 1$ configuration.

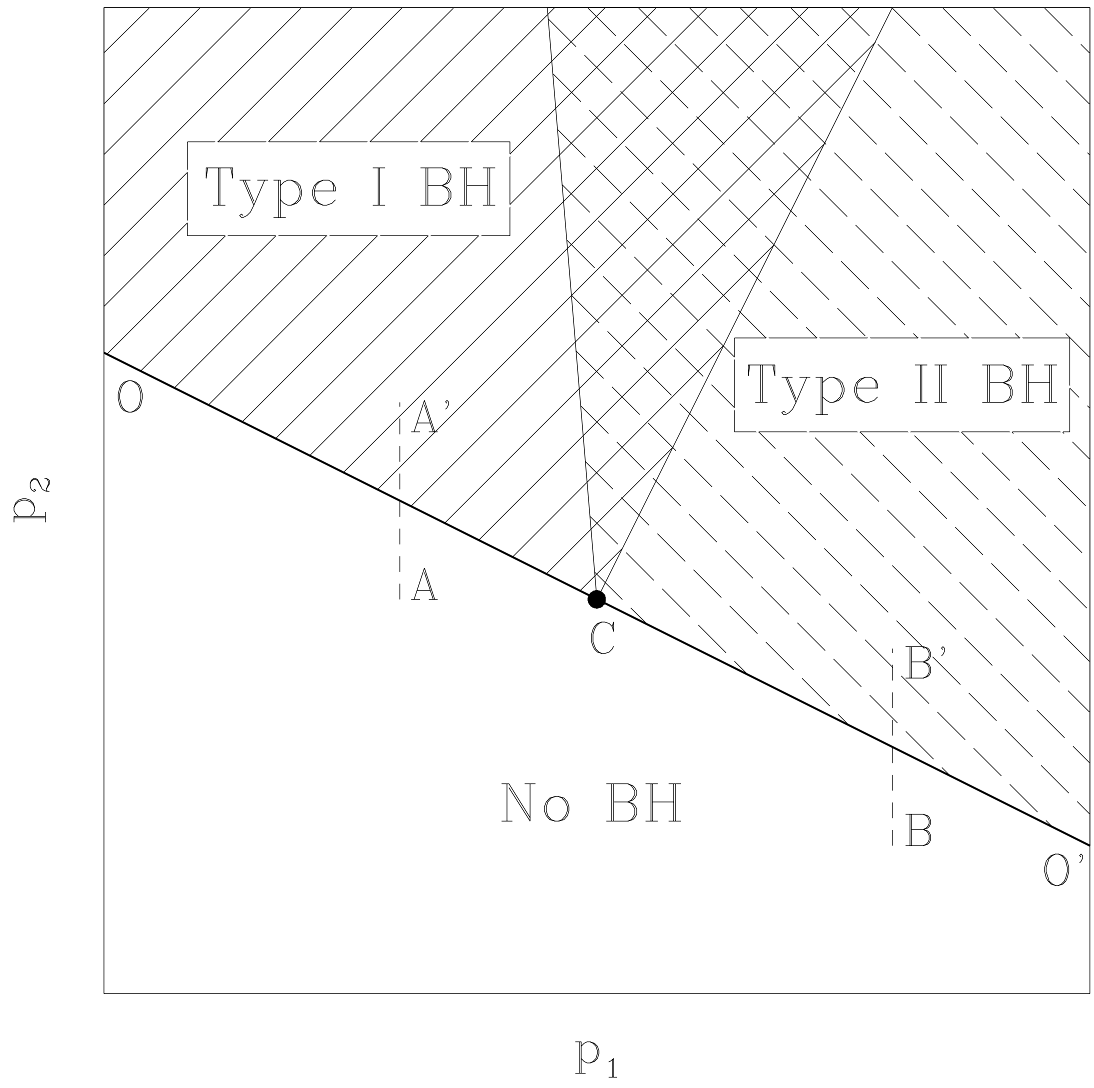
  - **Type II behaviour**: As $p \to p^\star$, dynamics asymptotes to discretely self-similar solution; black hole masses satisfy a scaling law

    $$\ln M_{BH} \approx \gamma \ln|p - p^\star|$$

    where $\gamma \approx 0.20$ is the reciprocal Lyapunov exponent of the single unstable mode for the DSS "intermediate attractor".

- Found that suitable *two* parameter families of initial data could generate both types of behaviour, and that transition from Type I to Type II was characterized by co-existence of the "phases".

Phase-Space Schematic for Magnetic EYM Collapse



- The two types of threshold behaviour co-exist at C.

- Similar picture also established for *massive* scalar collapse (**Brady, Chambers & Goncalves (1997)**)

Unanswered Questions

- What is going on in black hole regime? (“gaps” / “jumps” in mass spectra were observed)

- What happens to the purely magnetic critical solutions in the context of the *general* $(\tilde{w}, u, v \neq 0)$ spherically-symmetric model?

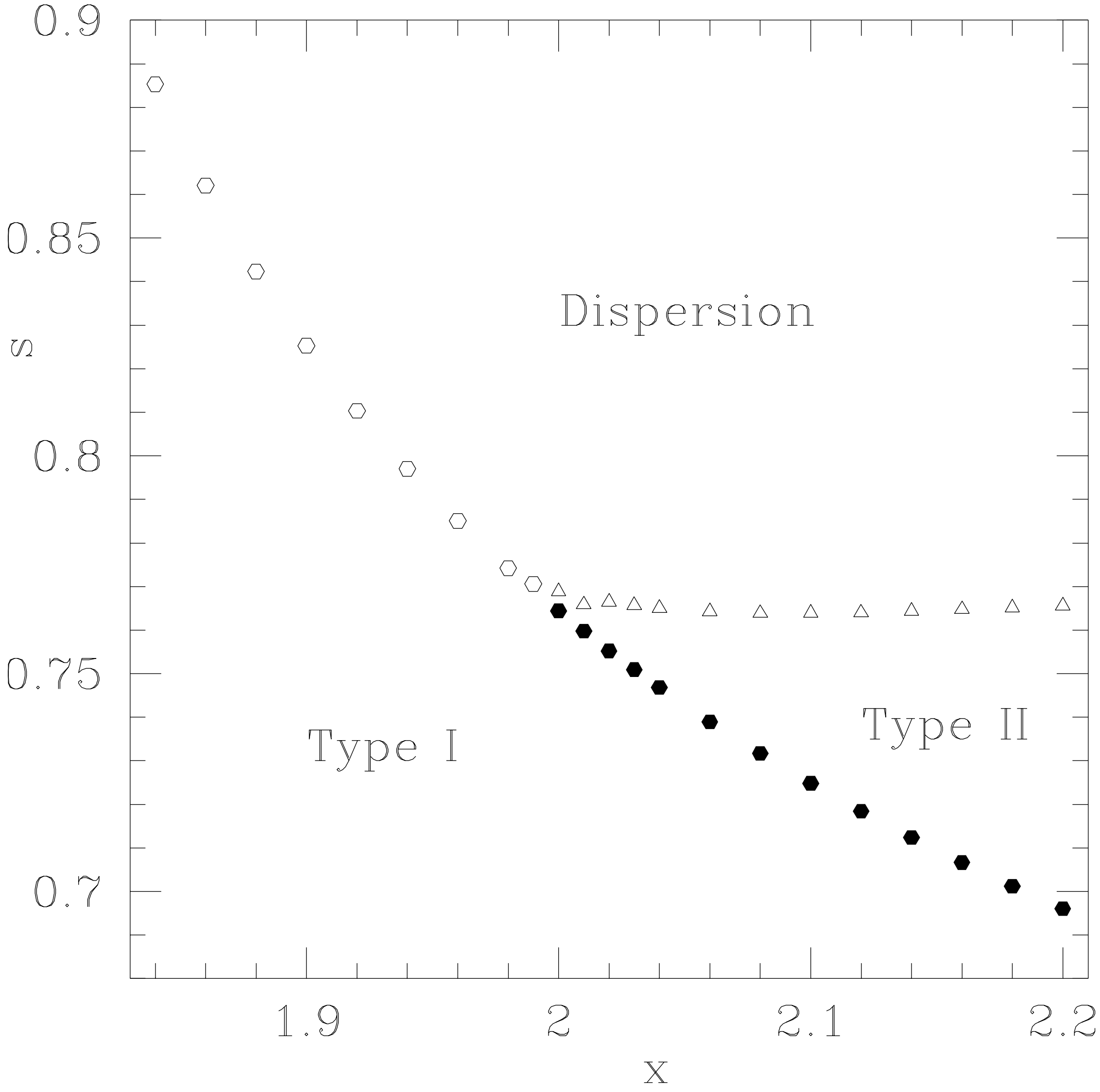
Forming Colored Black Holes via Fine Tuning

(Choptuik, Hirschmann and Marsa (gr-qc/9903081))

- Still work in magnetic ansatz, but now use maximal/areal coordinates and black-hole excision techniques so that BH regime can be efficiently studied

- Find new type of critical behaviour: as $p \to p^\star$, dynamics asymptotes to an $n = 1$ colored black hole, lifetime is again (presumably) related to the reciprocal Lyapunov exponent of the single unstable mode

New Phase-Space Schematic for Magnetic EYM Collapse



Instability of Magnetic Critical Solutions
(`Choptuik and Hirschmann (in progress)`)

- Now work within most general spherically symmetric framework: effectively have *two* independent potentials:

$$w(r,t) \qquad \tilde{w}(r,t)$$

  use polar/areal coordinates (no excision techniques)

- If $\tilde{w}(r,0) = 0$, then $\tilde{w}(r,t) = 0$: Start with initial data $w(r,0;p)$ which generates the $n = 1$ Barnik-Mckinnon solution at threshold ($\tilde{w}(r,0) = 0$), then "perturb" by setting $\tilde{w}(r,0) \neq 0$ and "re-tune" to critical point

- Perturbation theory suggests than $n = 1$ Barnik-Mckinnon solution will no longer be the threshold solution since there are now *two* unstable modes associated with that configuration

- Numerical results confirm this expectation—threshold solution is no longer static, but discretely (?) self-similar—somewhat surprisingly, it is apparently *not* the DSS solution found in the context of the purely magnetic ansatz

- Thus, Type I magnetic solution is unstable to addition of second potential; stability of Type II magnetic solution remains an open question

The Future

- Relaxing spherical symmetry
  - Axisymmetry
    (with **Hirschmann & Liebling** and **Garfinkle**)
  - Full 3D
    (could come along relatively quickly—ideal test application for 3D AMR)

- Other matter models

- Quantum effects
  - Do Type II solutions *demand* to be quantized?
  - What does the semi-classical approximation really say about critical behaviour
  - Does BH threshold game make sense in context of contemporary QG theories (i.e. strings, quantum geometry), and if so, what are predictions?

- Investigation/exploitation of phenomenology/analytic approaches from condensed matter theory (**Goldenfeld, Gundlach**)

The Future (cont.)

- *Please remember the deliverables/expertise.*

- I'm eager to work with other program members to upgrade/develop HPC at *all* levels (UBC/BC/Canada)