

```
> readlib(grii);
```

proc() end

```
> grtensor();
```

GRTensorII Version 1.70 (R5)

31 May 1998

Developed by Peter Musgrave, Denis Pollney and Kayll Lake

Copyright 1994-1998 by the authors.

Latest version available from: <http://astro.queensu.ca/~grtensor/>

```
> qload(newkn);
```

```
Error, (in grload) Metric name has already been used:, newkn
```

```
> ?grOptionMetricPath;
```

```
> \. \;
```

```
> R(dn, dn);
```

R(dn, dn)

```
> grcalc(R(dn, dn));
```

CPU Time = .002

```
> grdisplay(_);
```

For the newkn spacetime:

Covariant Ricci

R(dn, dn)

$R_{a b} =$

$$\left[-\frac{Q^2}{(r^2 + u^2)(r^2 - 2mr + a^2 + Q^2)}, 0, 0, 0 \right]$$

$$\left[0, \frac{Q^2}{(r^2 + u^2)(a^2 - u^2)}, 0, 0 \right]$$

$$\left[0, 0, ((r^2 u^4 - 2u^4 m r + a^2 u^4 + u^4 Q^2 + 4a^2 u^2 m r - 2a^2 u^2 Q^2 - 3a^4 u^2 - 4a^2 r^2 u^2 - r^4 u^2$$

$$+ 2a^6 + a^4 Q^2 + a^2 r^4 - 2a^4 m r + 3a^4 r^2) Q^2) / ((r^2 + u^2)^3 a^2),$$

$$- \frac{(-2r^2 u^2 - u^2 Q^2 + 2u^2 m r - 2a^2 u^2 + a^2 Q^2 + 2a^4 - 2a^2 m r + 2a^2 r^2) Q^2}{(r^2 + u^2)^3 a} \right]$$

$$\left[0, 0, -\frac{(-2r^2 u^2 - u^2 Q^2 + 2u^2 m r - 2a^2 u^2 + a^2 Q^2 + 2a^4 - 2a^2 m r + 2a^2 r^2) Q^2}{(r^2 + u^2)^3 a}, \right]$$

$$\left[\frac{Q^2 (-u^2 + 2a^2 + Q^2 - 2mr + r^2)}{(r^2 + u^2)^3} \right]$$

> grcalc (R(up, up, dn, dn));

CPU Time = .001

> grdisplay(_);

For the newkn spacetime:

Mixed Riemann

$$R^r u_{r u} = \frac{-m r^3 + r^2 Q^2 + 3 u^2 m r - u^2 Q^2}{(r^2 + u^2)^3}$$

$$R^r u_{\phi t} = \frac{(r^2 - 2mr + a^2 + Q^2)(a^2 - u^2)(u^2 m + 2rQ^2 - 3mr^2)u}{(r^2 + u^2)^4 a}$$

$$R^r \phi_{r \phi} = (9a^2 u^2 m r - 3a^2 r^3 m + 4a^2 r^2 Q^2 - 2a^2 u^2 Q^2 + r^4 Q^2 - 6u^4 m r - 4r^2 u^2 Q^2 - r^5 m + u^4 Q^2 + 5r^3 u^2 m) / (r^2 + u^2)^4$$

$$R^r \phi_{r t} = -\frac{a(9u^2 m r - 3m r^3 + 4r^2 Q^2 - 2u^2 Q^2)}{(r^2 + u^2)^4}$$

$$R^r \phi_{u \phi} = u(-9m a^2 r^2 + 3m a^2 u^2 + 6r a^2 Q^2 - 2m^2 r u^2 + Q^2 u^2 m + 3m r^2 u^2 + 2Q^4 r - 9r^4 m - 7m r^2 Q^2 + 6m^2 r^3 + 6r^3 Q^2) / (r^2 + u^2)^4$$

$$R^r \phi_{u t} = -((-2m u^4 + 3m a^2 u^2 + 7m r^2 u^2 - 2m^2 r u^2 + Q^2 u^2 m - 4r u^2 Q^2 - 7m r^2 Q^2 + 6m^2 r^3 - 3r^4 m + 2r^3 Q^2 - 9m a^2 r^2 + 6r a^2 Q^2 + 2Q^4 r) a u) / ((a^2 - u^2)(r^2 + u^2)^4)$$

$$R^r t_{r \phi} = ((a^2 - u^2)(9a^2 u^2 m r - 3a^2 r^3 m + 4a^2 r^2 Q^2 - 2a^2 u^2 Q^2 + 9r^3 u^2 m - 3r^5 m + 4r^4 Q^2 - 2r^2 u^2 Q^2)) / ((r^2 + u^2)^4 a)$$

$$R^r t_{r t} = -(9a^2 u^2 m r - 3a^2 r^3 m + 4a^2 r^2 Q^2 - 2a^2 u^2 Q^2 + 3r^4 Q^2 - 3u^4 m r - 2r^2 u^2 Q^2 - 2r^5 m + u^4 Q^2 + 7r^3 u^2 m) / (r^2 + u^2)^4$$

$$R^r t_{u \phi} = ((-Q^2 m u^4 - u^4 r^2 m - a^2 u^4 m + 2m^2 r u^4 + 5r^4 u^2 m + a^2 Q^2 u^2 m - 2r^3 u^2 Q^2 + 8a^2 r^2 u^2 m - 2a^2 m^2 r u^2 - 6m^2 r^3 u^2 - 2a^2 u^2 r Q^2 - 2Q^4 r u^2 + 7r^2 Q^2 u^2 m + 3a^4 m u^2 + 4r^5 Q^2 - 6r^6 m + 10a^2 r^3 Q^2 - 7a^2 m r^2 Q^2 + 2a^2 Q^4 r + 6a^4 r Q^2 - 15a^2 r^4 m + 6a^2 m^2 r^3 - 9a^4 m r^2) u) / ((r^2 + u^2)^4 a)$$

$$R^{r^t}_{u^t} = -u(-9m^2r^2 + 3m^2u^2 + 6ra^2Q^2 - 2m^2ru^2 + Q^2u^2m + 3mr^2u^2 + 2Q^4r - 9r^4m - 7mr^2Q^2 + 6m^2r^3 + 6r^3Q^2) / (r^2 + u^2)^4$$

$$R^{u\phi}_{r\phi} = ((-3a^2u^4m + 4m^2ru^4 - 2Q^2mu^4 - 3u^4r^2m - 12m^2r^3u^2 - 4a^2m^2ru^2 + 2a^2Q^2u^2m - 6a^2u^2rQ^2 - 4Q^4ru^2 + 14r^2Q^2u^2m - 6r^3u^2Q^2 + 12a^2r^2u^2m + 9r^4u^2m + 3a^4mu^2 + 4a^2Q^4r - 9a^4mr^2 - 14a^2mr^2Q^2 + 6a^4rQ^2 + 12a^2m^2r^3 - 9a^2r^4m + 6a^2r^3Q^2)u) / ((r^2 - 2mr + a^2 + Q^2)(r^2 + u^2)^4)$$

$$R^{u\phi}_{r^t} = -((-m^4u^4 + 3m^2a^2u^2 + 5m^2r^2u^2 - 4m^2ru^2 + 2Q^2u^2m - 2ru^2Q^2 - 14mr^2Q^2 + 12m^2r^3 - 6r^4m + 4r^3Q^2 - 9ma^2r^2 + 6ra^2Q^2 + 4Q^4r)au) / ((r^2 - 2mr + a^2 + Q^2)(r^2 + u^2)^4)$$

$$R^{u\phi}_{u\phi} = -(9a^2u^2mr - 3a^2r^3m + 2a^2r^2Q^2 - 4a^2u^2Q^2 + r^4Q^2 - 3u^4mr - 4r^2u^2Q^2 - 2r^5m + u^4Q^2 + 7r^3u^2m) / (r^2 + u^2)^4$$

$$R^{u\phi}_{u^t} = \frac{a(9u^2mr - 3mr^3 + 2r^2Q^2 - 4u^2Q^2)}{(r^2 + u^2)^4}$$

$$R^{u^t}_{r\phi} = ((-2a^2u^4m - 2u^4r^2m - 2Q^2mu^4 + 4m^2ru^4 - 4r^3u^2Q^2 + 7r^4u^2m - 4a^2u^2rQ^2 - 4a^2m^2ru^2 - 12m^2r^3u^2 + 2a^2Q^2u^2m + 3a^4mu^2 + 14r^2Q^2u^2m - 4Q^4ru^2 + 10a^2r^2u^2m - 14a^2mr^2Q^2 + 4a^2Q^4r - 12a^2r^4m + 6a^4rQ^2 - 9a^4mr^2 + 8a^2r^3Q^2 + 2r^5Q^2 - 3r^6m + 12a^2m^2r^3)(a^2 - u^2)u) / (a(r^2 - 2mr + a^2 + Q^2)(r^2 + u^2)^4)$$

$$R^{u^t}_{r^t} = -((3m^2a^2u^2 + 3m^2r^2u^2 - 4m^2ru^2 + 2Q^2u^2m - 14mr^2Q^2 + 12m^2r^3 - 9r^4m + 6r^3Q^2 - 9ma^2r^2 + 6ra^2Q^2 + 4Q^4r)(a^2 - u^2)u) / ((r^2 - 2mr + a^2 + Q^2)(r^2 + u^2)^4)$$

$$R^{u^t}_{u\phi} = -((a^2 - u^2)(9a^2u^2mr - 3a^2r^3m + 2a^2r^2Q^2 - 4a^2u^2Q^2 + 9r^3u^2m - 3r^5m + 2r^4Q^2 - 4r^2u^2Q^2)) / (a(r^2 + u^2)^4)$$

$$R^{u^t}_{u^t} = (9a^2u^2mr - 3a^2r^3m + 2a^2r^2Q^2 - 4a^2u^2Q^2 + r^4Q^2 - 6u^4mr - 2r^2u^2Q^2 - r^5m + 3u^4Q^2 + 5r^3u^2m) / (r^2 + u^2)^4$$

$$R^{\phi t}_{r^u} = -\frac{a(u^2m + 2rQ^2 - 3mr^2)u}{(r^2 + u^2)^2(a^2 - u^2)(r^2 - 2mr + a^2 + Q^2)}$$

□ >

$$R^{\phi_t}_{\phi_t} = \frac{-m r^3 + r^2 Q^2 + 3 u^2 m r - u^2 Q^2}{(r^2 + u^2)^3}$$