PHY 387M: Relativity Theory I   Fall 1997   Homework 3
Due: Tuesday, October 14, 12:30pm

Problem 1 (Wald 3.3):
a) Show that

$$R_{abcd} = R_{cdab}\,.$$

b) In n dimensions, the Riemann tensor (here defined using the derivative operator ∇_a naturally associated with the metric, $\nabla_a g_{bc} = 0$) has n^4 components. However, on account of the symmetries

$$\begin{aligned} R_{abcd} &= -R_{bacd}\,, \\ R_{abcd} &= -R_{abdc}\,, \\ R_{[abc]d} &= 0, \end{aligned}$$

not all of these components are independent. Show that the number of independent components is

$$\frac{1}{12} n^2 \left(n^2 - 1\right)\,.$$

Problem 2 (Wald 3.5):
a) Show that any curve whose tangent satisfies

$$T^a \nabla_a T^b = \alpha T^b\,,$$

where α is an arbitrary function on the curve, can be reparameterized so that

$$T^a \nabla_a T^b = 0$$

is satisfied.

b) Let t be an affine parameter of a geodesic γ. Show that all other affine parameters of γ take the form $at + b$, where a and b are constants.

Problem 3 (Wald 3.6b): The metric of Euclidean $\mathbf{R}^3$ in spherical coordinates, (r, θ, ϕ), is

$$ds^2 = dr^2 + r^2 \left(d\theta^2 + \sin^2\theta\, d\phi^2\right)$$

(see Homework 1, Problem 6 and Homework 2, Problem 4). Using the Christoffel components, $\Gamma^\sigma{}_{\mu\nu}$, you computed in Homework 2, write down the components of the geodesic equation in this coordinate system, and verify that the solutions correspond to straight lines in Cartesian coordinates.

Problem 4 (Wald 3.7a): An arbitrary Lorentz metric on a two-dimensional manifold locally always can be put in the form

$$ds^2 = \Omega^2(x,t)\left(-dt^2 + dx^2\right)\,.$$

Calculate the components of the Riemann curvature tensor of this metric (by hand), using the coordinate basis methods discussed in class and described in section 3.4a of Wald.

Problem 5: Using `Maple` and `GRTensor`, verify that all of the components, $R_{\mu\nu\rho\sigma}$, of the Riemann tensor vanish for the flat spacetime metric expressed in the "rotating coordinates" of Homework 2, Problem 1. If you use `xmaple`, print out and submit a postscript version of a worksheet showing the calculations. If you use `maple`, use the Unix `script` command (type `man script` for information) to record your session (as ASCII text) to a file, then print out and submit hardcopy of the file using the `lpr` command. Please refer to the on-line course notes (via the course Web page) concerning the use of the Center for Relativity computers, `Maple` and `GRTensor`, and consult the instructor if you encounter any problems.