PHY 387M: Relativity Theory I Fall 1997 Homework 5
Due: Tuesday, December 2, 12:30pm

Problem 1 (Wald 6.1): Let M be a three-dimensional manifold possessing a spherically symmetric Riemannian metric with $\nabla_a r \neq 0$, where r is defined by

$$r = (A/4\pi)^{1/2}$$

(A is the total area of the constant-r 2-sphere).

a) Show that a new "isotropic" radial coordinate $\tilde{r}$ can be introduced so that the metric takes the form

$$ds^2 = H(\tilde{r})\left[d\tilde{r}^2 + \tilde{r}^2\, d\Omega^2\right]$$

(This shows that every spherically symmetric three-dimensional space is conformally flat.)

b) Show that in isotropic coordinates the Schwarzshild metric is

$$ds^2 = -\frac{\left(1 - M/\left(2\tilde{r}\right)\right)^2}{\left(1 + M/\left(2\tilde{r}\right)\right)^2} dt^2 + \left(1 + \frac{M}{2\tilde{r}}\right)^4 \left[d\tilde{r}^2 + \tilde{r}^2\, d\Omega^2\right]$$

Problem 2 (Wald 5.4a) Let (M, g_{ab}) be a stationary spacetime with timelike Killing vector field ξ^a. Let $V^2 = -\xi^a \xi_a$. Show that the acceleration $a^b = u^a \nabla_a u^b$ of a stationary observer is given by

$$a^b = \nabla^b \ln V$$