Maximum mass and maximum collapse function for Boson Star Models

Physics 555b Term Project

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Outline

- Introduction to Boson stars (BS)
- Motivation
- Formalism
- Choice of BS models
- The massive boson stars
- Maximum mass versus coupling coefficient
- Quick example
- Collapse function
- Future work

Introduction to Boson Star (BS)

- What is a boson star?
  - Historical perspective
    - Wheeler (1955): GEONS, electromagnetic self-gravitating entities
    - Kaup (1968): Klein-Gordon geons, a minimally coupled massive complex scalar field to general relativity (rather than the EM field)
    - Ruffini and Bonazzola (1969): showed that the classical limit for the BS stress-energy tensor could be obtained by the mean value of its quantum counterpart over the ground state vector for a system of many particles. At zero temperature, a large fraction of the total number of bosons in the system will occupy this ground state (BEC). The link between the quantum mechanics treatment of bosons and the classical view of scalar fields were then established.
  - BS is a self-gravitating compact object (compact in the sense that its radius is of the order of Schwarzschild radius) composed of a large number of scalar particles in their ground state (BEC), described classically by a complex scalar field minimally coupled to gravity.

Introduction to Boson Star (BS)

- Why are BS interesting?
  - Particle interest: there exists no known fundamental scalar particle up to date. BS would be then our cosmological lab.
  - Cosmological/Astrophysical interests:
    - Inflation field is a scalar field. Stars resulting from those fields may have played an important role in the inflationary period.
    - Scalar particles have been proposed as a good candidate for the dark matter in the universe.
    - Since boson stars could achieve a very large size, they could offer an alternative to super black holes in galactic centers. Boson stars should exhibit distinct lensing effects of which could be helpful in its detection and determination of its properties.
    - Certainly the studies of the collapse of such a boson cloud of scalar particle into boson star would lead to a better understanding of the astrophysical phenomena.
  - OUR MAIN INTEREST: to investigate the strong gravitational field regime through numerical relativity.

Motivation

- Why are scalar fields a tempting matter model candidate for studying the strong field regime?
  - A massive complex field is chosen as matter source because it is a simple type of matter that allows a star-like solution and because there will be no problems with shocks, low density regions, ultrarelativistic flows, etc in the evolution of this kind of matter as opposed to fluids
  - The fermionic and bosonic system share some general features from the beginning: for example, in spherical symmetry we can parameterize the family of solutions by the modulus of the field at $r = 0$, the central field, ϕ_0, which is analogous to the central density for perfect fluid stars.
  - It's a good candidate for studying systems where the details of the dynamics of the stars (e.g. shocks) tend not to be important gravitationally, as for example, in binaries of compact stars. Boson star binaries then may provide some insight into neutron star ones.

Motivation

- What properties of boson stars are we going to focus on?
  - Short answer: Their gravitational equilibrium described by two parameters: the maximal stellar mass and its collapse function.
    - Boson stars are prevented from collapsing gravitationally by the pressure stemmed from the Heisenberg uncertainty principle. Like their fermion counterparts, neutron stars and white dwarves, boson stars also have a limiting ADM mass below which the star is stable against complete gravitational collapse into a black hole (BH).
    - As for the neutron star case (where the Pauli exclusion priciple provides the degeneracy pressure), we can also derive an expression for the maximum possible mass. This turns out to be $\sim M_{pl}^3/m^2$, where M_{pl} is the planck mass and m the scalar field mass, while the maximum mass of a non-self-interacting boson star is $\sim M_{pl}^2/m$.
    - Colpi et al. added a self-interacting potential of the form $\lambda|\phi^4|$. Their results showed actually that a BS could have a size and mass of the order of their fermionic counterpart $\sim \lambda^{1/2} M_{pl}^3/m^2$.

Motivation

- OUR MAIN MOTIVATION 1: generalize the work done by Colpi at al. for other preferably fancy potentials and look for some interesting behaviour for the maximal stellar mass as the parameters (couplings) of the model changes.

- OUR MAIN MOTIVATION 2: answer the question: what is the most compact stable boson star model?

  - For a fluid star, the schwarzschild limit is defined as the minimum coordinate radius that a mass can have under static equilibrium. It is well know result coming from the search for possible interior fluid sources for an external Schwarzschild spacetime solution.

  - It can be described by its collapse function, that for a static configuration of a fluid star is bounded to $8/9$:

$$z = \frac{2m(r)}{r} \leq 8/9 \tag{1}$$

  - We would like to find then a analogous upper bound for boson stars (apart from the obvious $z \leq 1$) for a class of different types of self-interacting potencials and obtain a map of the the maximum z as a function of the coupling coefficients for a particular self-interaction potential.

  - This value would correspond then to the most compact static, stable boson star

Formalism

We begin with an Einstein-Klein-Gordon system with a self-interaction potential

$$\mathcal{L}_\phi = 1\frac{1}{2}\left(\nabla^\mu\phi\nabla_\mu\phi^* + U(|\phi|^2\right)$$

$$T_{\mu\nu} = \frac{1}{2}\left[(\nabla_\mu\phi\nabla_\nu\phi^* + \nabla_\nu\phi\nabla_\mu\phi^*) - g_{\mu\nu}\left(\nabla^\alpha\phi\nabla_\alpha\phi + U(|\phi|^2\right)\right]$$

$$\nabla^\mu\nabla_\mu\phi = \frac{dU(|\phi|^2)}{d|\phi|^2}\phi$$

The metric used

- In a spherically symmetric spacetime with timelike Killing vectors, the metric is of the form

$$ds^2 = -\alpha^2 dt^2 + a^2 dr^2 + r^2 b^2 d\Omega^2$$

The Choice of Slicing

- We choose the polar slicing and the areal coordinate condition.

$$K = K^r{}_r \quad \text{and} \quad b = 1$$

Formalism

The Ansatz

- We demand that the spacetime be dependent.
- Complex scalar field must then be on the form

$$\phi(r,t) = \phi_0(r)e^{-i\omega t}$$

The Equations of motion

For the system laid out we derive the equations of motion which form a set of ODE's

$$a' = \frac{1}{2}\left\{\frac{a}{r}(1-a^2) + 4\pi a r\left[a^2 U(\phi_0^2) + \frac{\omega^2}{\alpha^2}\phi_0^2 a^2 + \Phi_0^2\right]\right\}$$

$$\alpha' = \frac{\alpha}{2}\left\{\frac{1}{r}(a^2-1) + 4\pi a r\left[\frac{\omega^2}{\alpha^2}\phi_0^2 a^2 - a^2 U(\phi_0^2) + \Phi_0^2\right]\right\}$$

$$\phi_0' = \Phi_0$$

$$\Phi_0' = \left(\frac{dU(\phi_0^2)}{d\phi_0^2} - \frac{\omega^2}{\alpha^2}\right)a^2\phi_0 - \left(1 + a^2 - 4\pi r^2 a^2 U(\phi_0^2)\right)\frac{\Phi_0}{r}$$

Formalism

Defining the Mass of a Star

- Before starting to look for the maximum mass of the star, we need to define what we mean by mass.
- We define the mass of the star to be the ADM mass at infinity.
- At infinity, our metric approaches the Schwarzschild metric

$$ds^2 = -\left(1 - \frac{2M}{R}\right) dt^2 + \left(1 - \frac{2M}{R}\right)^{-1} dt^2 + r^2 d\Omega^2$$

- From the metric of our model, we can get the mass aspect function from the metric tensor function, $a(r)$, at infinity by comparing terms

$$m(r) = \frac{r}{2}\left(1 - \frac{1}{a^2}\right)$$

- We now get the ADM mass from the mass aspect function evaluated at infinity

$$M = \lim_{r \to \infty} m(r)$$

Formalism

Defining the Collapse Function (CF)

- One of the things that we are interested in is the compactness of a star
- The compactness of a star can be expressed in the fraction

$$z = \frac{2m(r)}{r}$$

- This fraction is called the collapse function
  - CF is the ratio of the Schwarzschild radius to the coordinate radius.
  - At an event horizon, $z = 1$.
- The compactness can now be easily found by $z = 1 - \frac{1}{a^2}$

Comparison With a Fluid Star

- As mentioned BS's are similar to FS's in many ways.
- FS's have an upper bound on CF: $z \leq \frac{8}{9}$

Choice of BS models

There is a great big sea of potentials for scalar fields to choose from. The bare minimum that we are interested in is the mass term

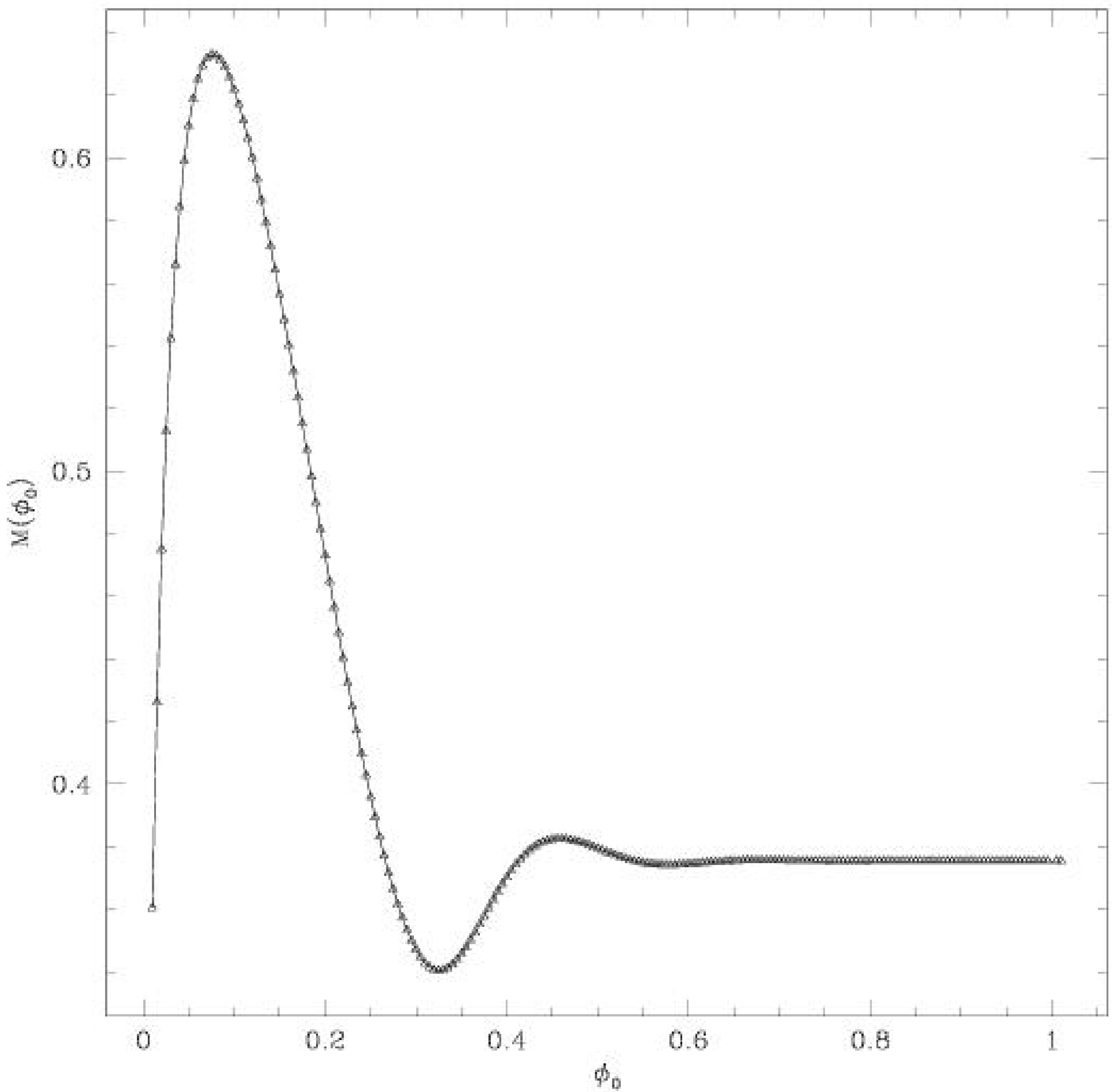
$$U(|\phi|^2) = m^2|\phi|^2$$

For further analysis, we considered potentials that would be added to the mass term

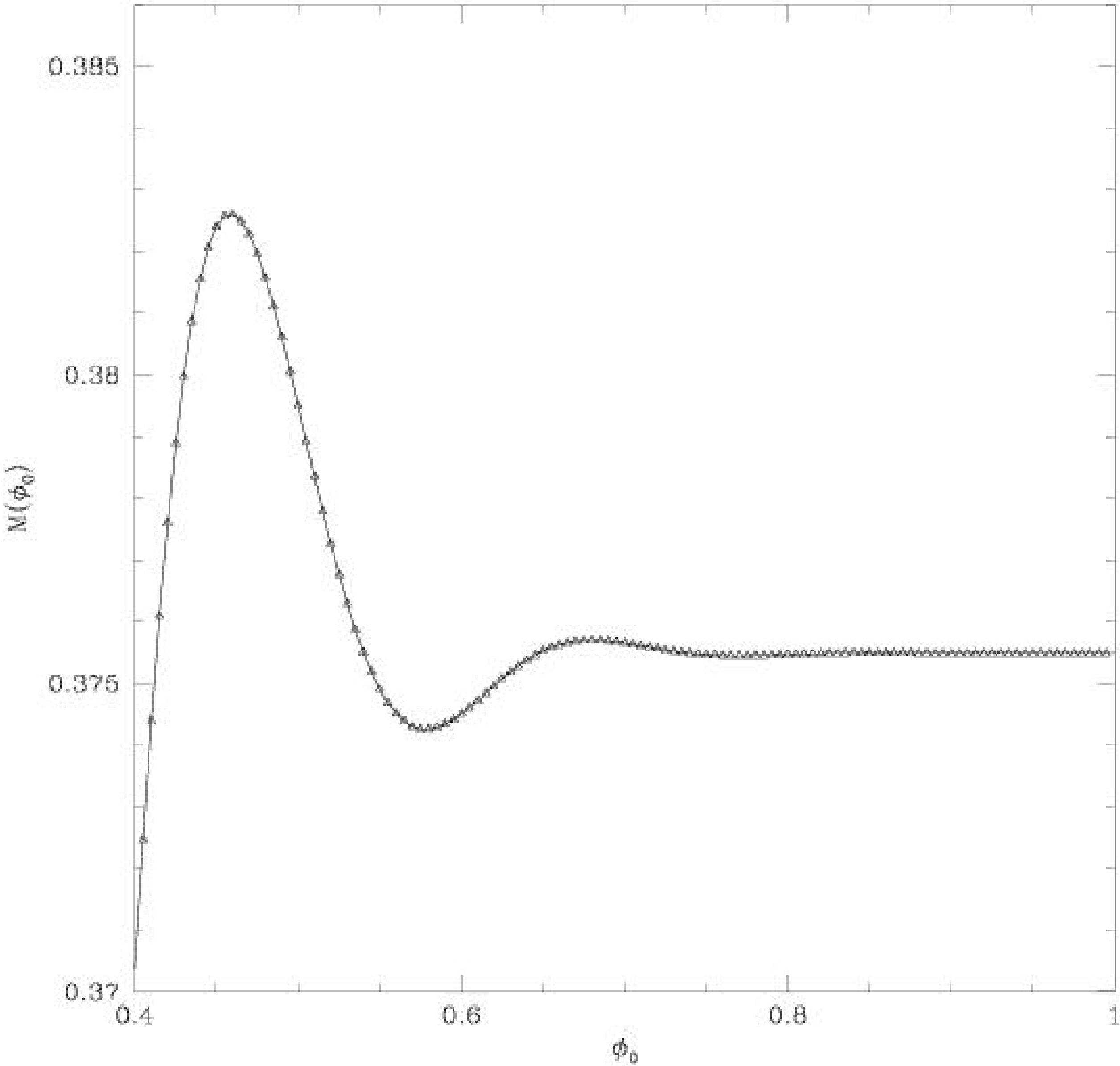
- We briefly investigated $\alpha m^2[\cosh(|\phi|) - 1]$ and $\alpha m^2[\sin(\frac{\pi}{2}[\beta|\phi| - 1]) + 1]$. Both worked, but didn't handle well as it was difficult to gain insight into them analytically.
- We also considered the dilaton case. Since the dilaton is a real field, it cannot obey our ansatz and is unable to carry charge.
- Whatever the interaction, it can be expanded as a power series in $|\phi|$.
- Our strategy is to look at several powers terms separately and then try to explore combinations of those.
- Investigation completed on the ϕ^3, ϕ^4 and ϕ^6 potentials
- Preliminary results for the $\phi^3 + \phi^4$ potential

Massive Boson Star

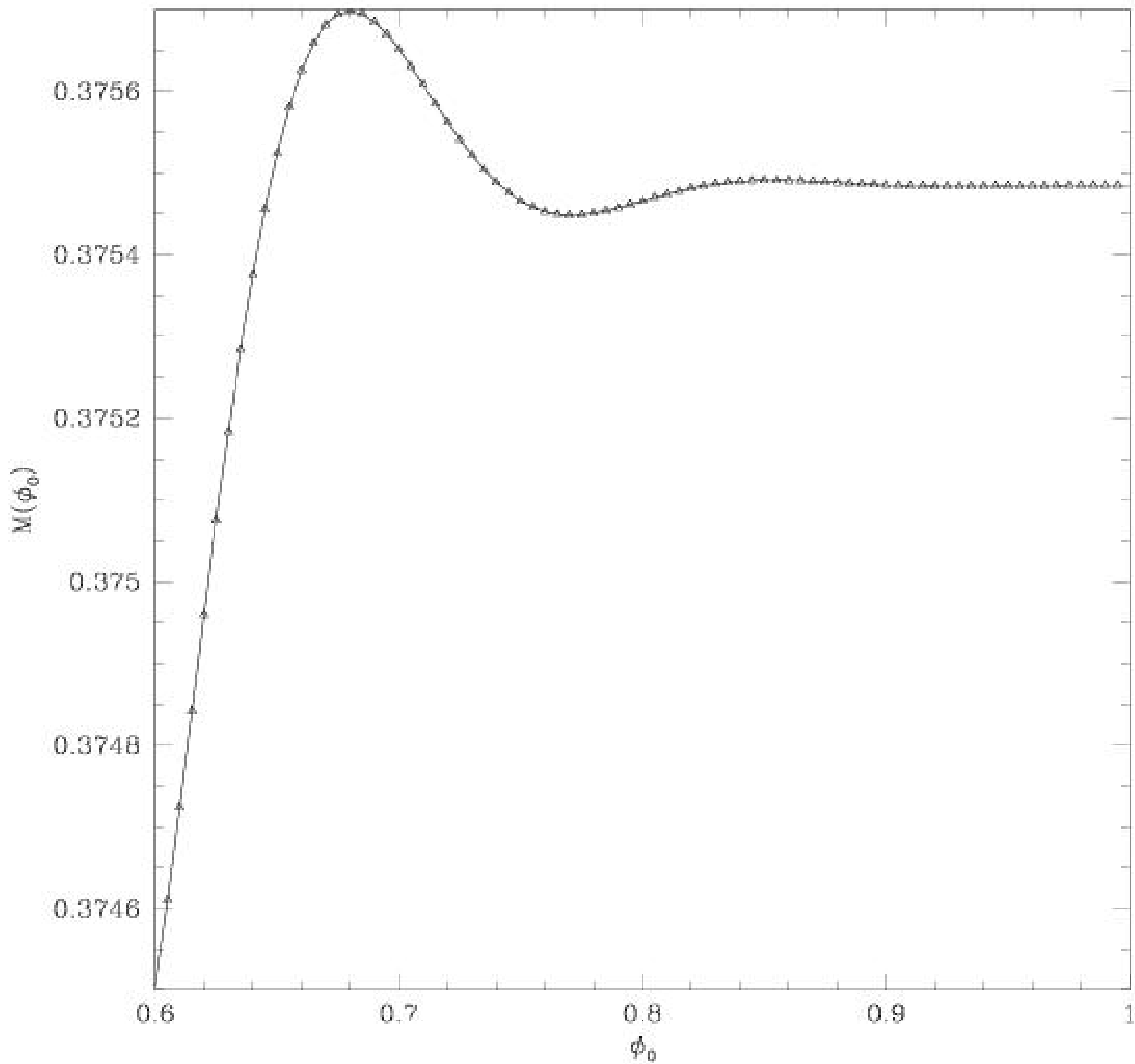
- Total Mass versus Central Field



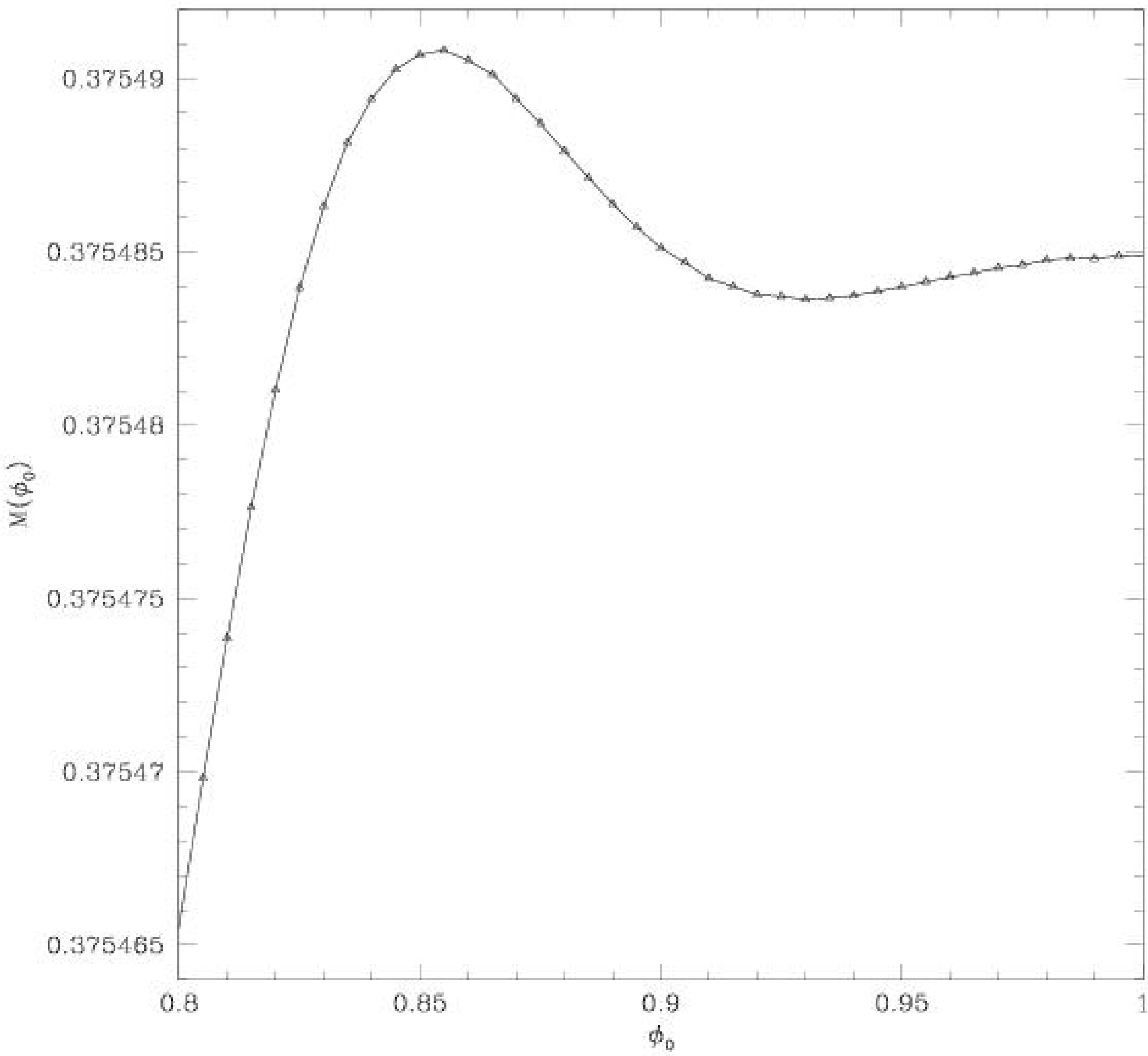
Massive Boson Star



Massive Boson Star



Massive Boson Star



Maximum Mass Versus Coupling Coefficient

- There is a known asymptotic relationship between the maximum mass of the stable boson star, and the coupling constant

$$R \sim 1/m \rightarrow M_{max} \sim M_{Pl}^2/m \tag{2}$$

- which is the relationship developed for a moderately relativistic massive KG boson star

- However if we consider a system with self interactions, and presumably independent coupling constants, we have a slightly more complicated form of the relation.

- The importance of the interaction potential is measured by the ratio

$$\frac{V(\phi)}{m^2|\phi|^2} \tag{3}$$

- V contains only the self interacting terms, no explicitly mass dependent terms

A quick example

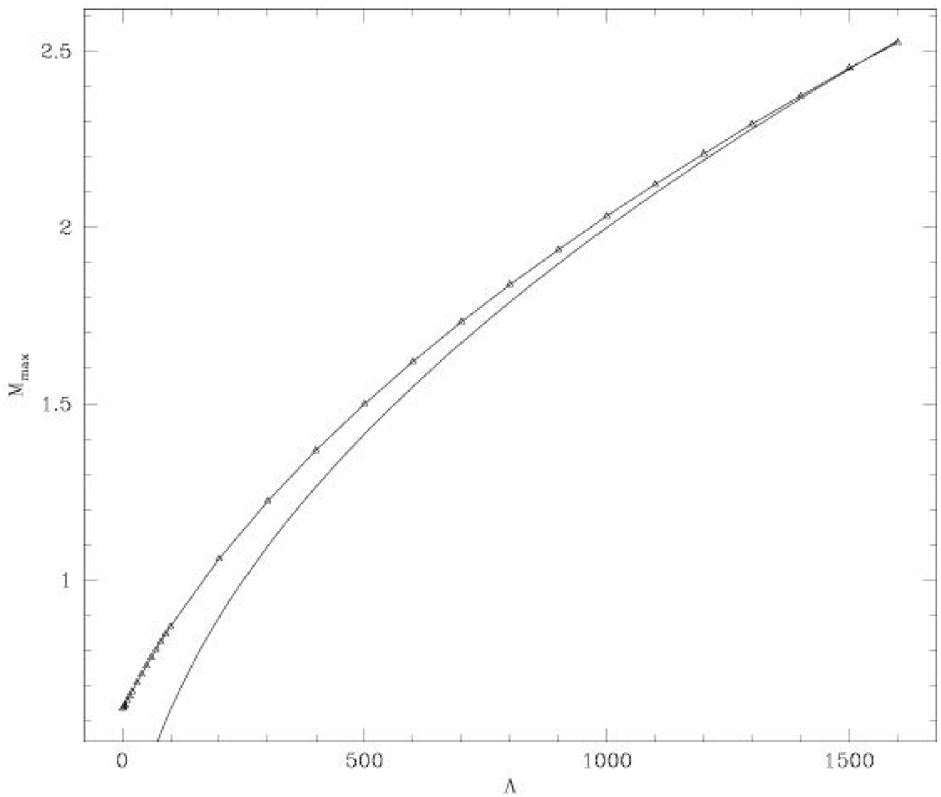
- So for say the ϕ^4 potential we have the ratio

$$\frac{\lambda|\phi|^4}{m^2|\phi|^2} \tag{4}$$

$$\frac{\lambda|\phi|^4}{m^2|\phi|^2} \sim \Lambda\frac{|\phi|^2}{M_{Pl}^2} \sim O(1) \tag{5}$$

- where we define $\Lambda = \frac{\lambda M_{Pl}^2}{m^2}$.
- Effectively we have rescaled the mass $m \to \Lambda^{-1/2}m$
- The radius is now $R_\lambda \sim \frac{Lambda^{1/2}}{m}$
- Then we are left with the field $|\phi| \sim m$
- With this we have that $M_\lambda^{max} \sim \Lambda^{1/2}\frac{M_{Pl}^2}{m}$ the asymptotic relationship for ϕ^4 theory.

Maximum Mass Versus Coupling Coefficient ϕ^4

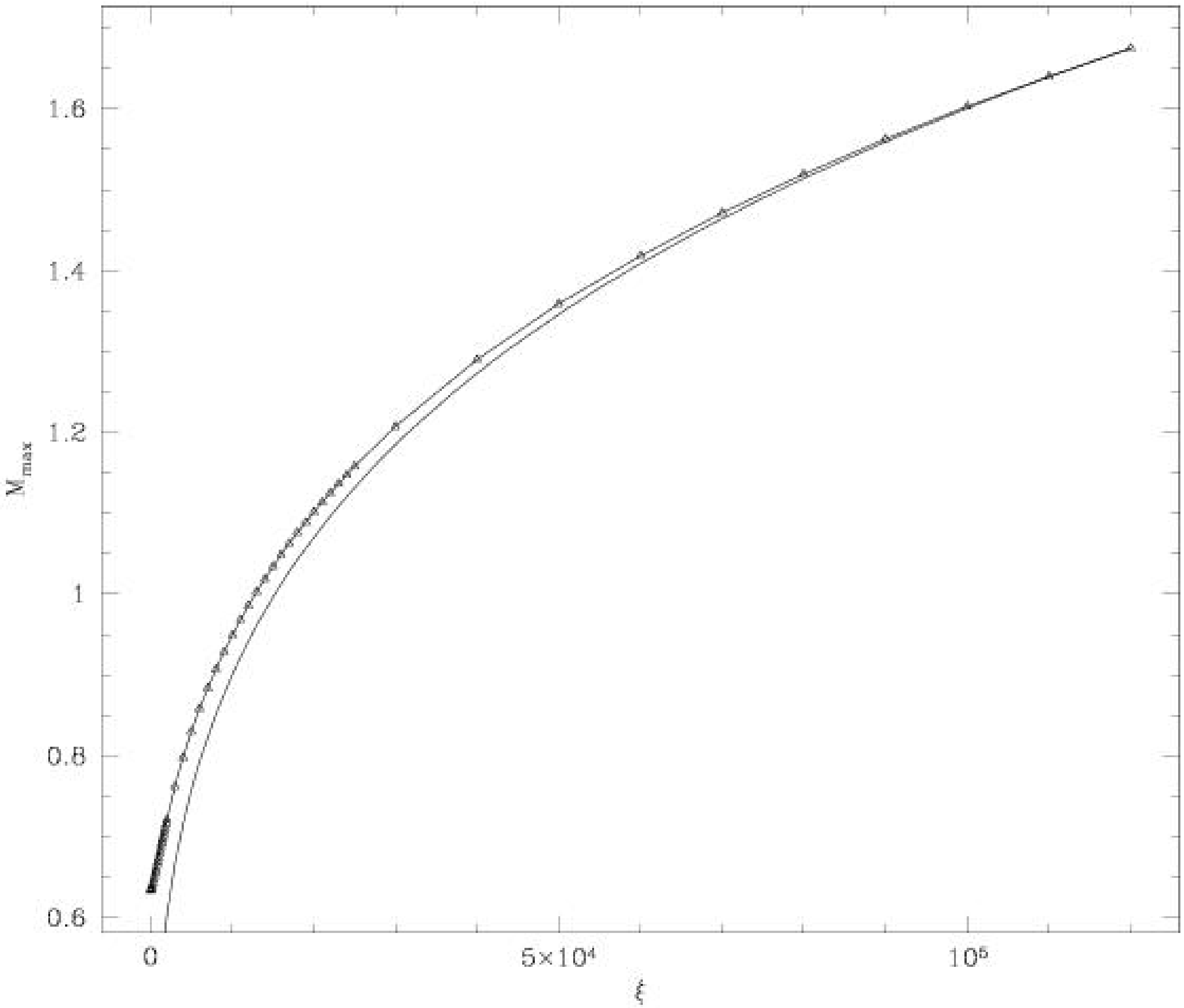


- the amplitude for the above relationship is 0.063, which agree exactly with Colpi's results

Maximum Mass Versus Coupling Coefficient

- Similarly we can have a maximum mass for a ϕ^6 theory

- With this we have that $M_\eta^{max} \sim \eta^{1/4} \frac{M_{Pl}^2}{m}$ the asymptotic relationship for ϕ^6 theory.



- here the amplitude for the above relationship was found to be 0.09

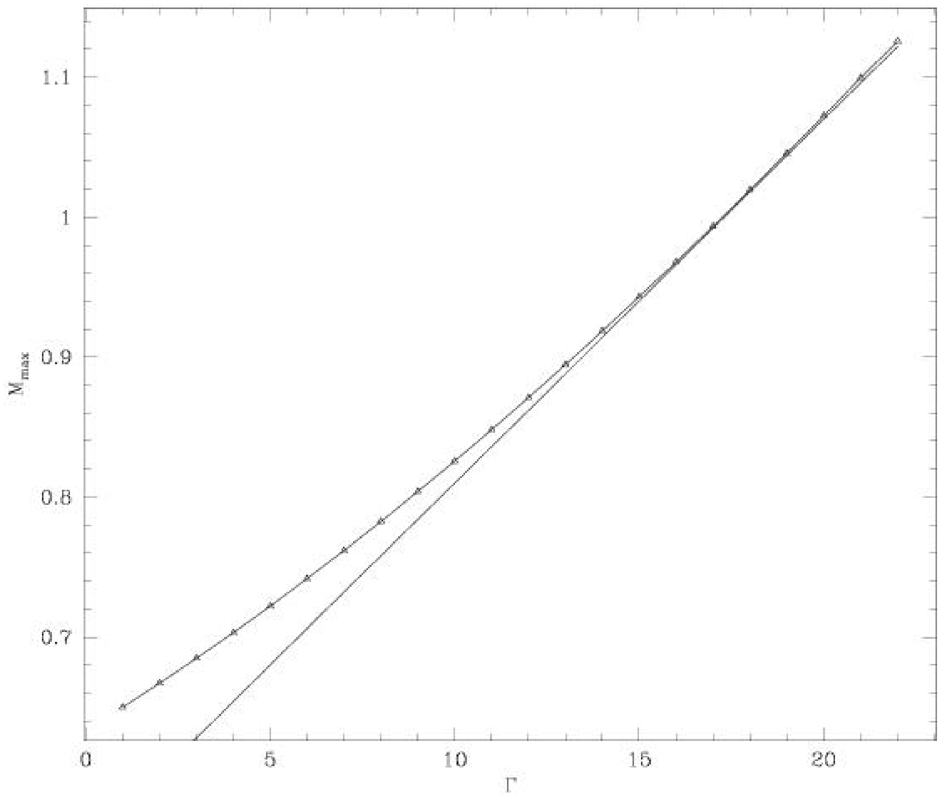
Maximum Mass Versus Coupling Constant

- For a general polynomial potential we have;

$$U(|\phi|) = \sum_{n=1}^{N} C_n |\phi|^n \tag{6}$$

- Since these Maximum mass relationships appear to have the trend $M^{max} \sim C_i^{1/(i-2)} \frac{M_{Pl}^2}{m}$
- when they are investigated without other interactions present. We thought we would investigate this for i=3 to find that we expect a linear asymptote.

Maximum Mass Versus Coupling Constant



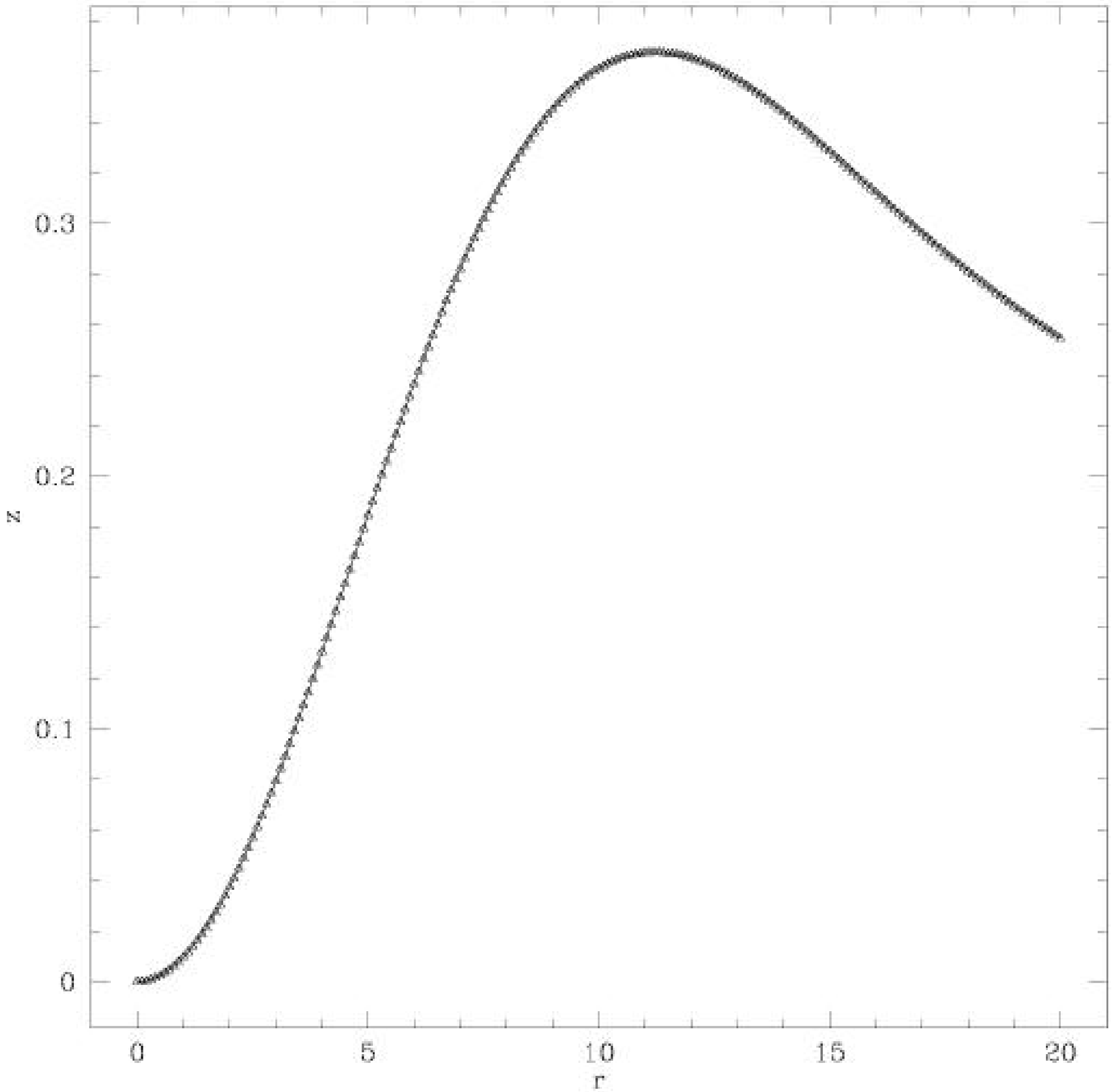
- the above asymptote has a slope of 0.025

Collapse Function

- For the second part of our investigation we look at the collapse function, $z = \frac{2m(r)}{r}$

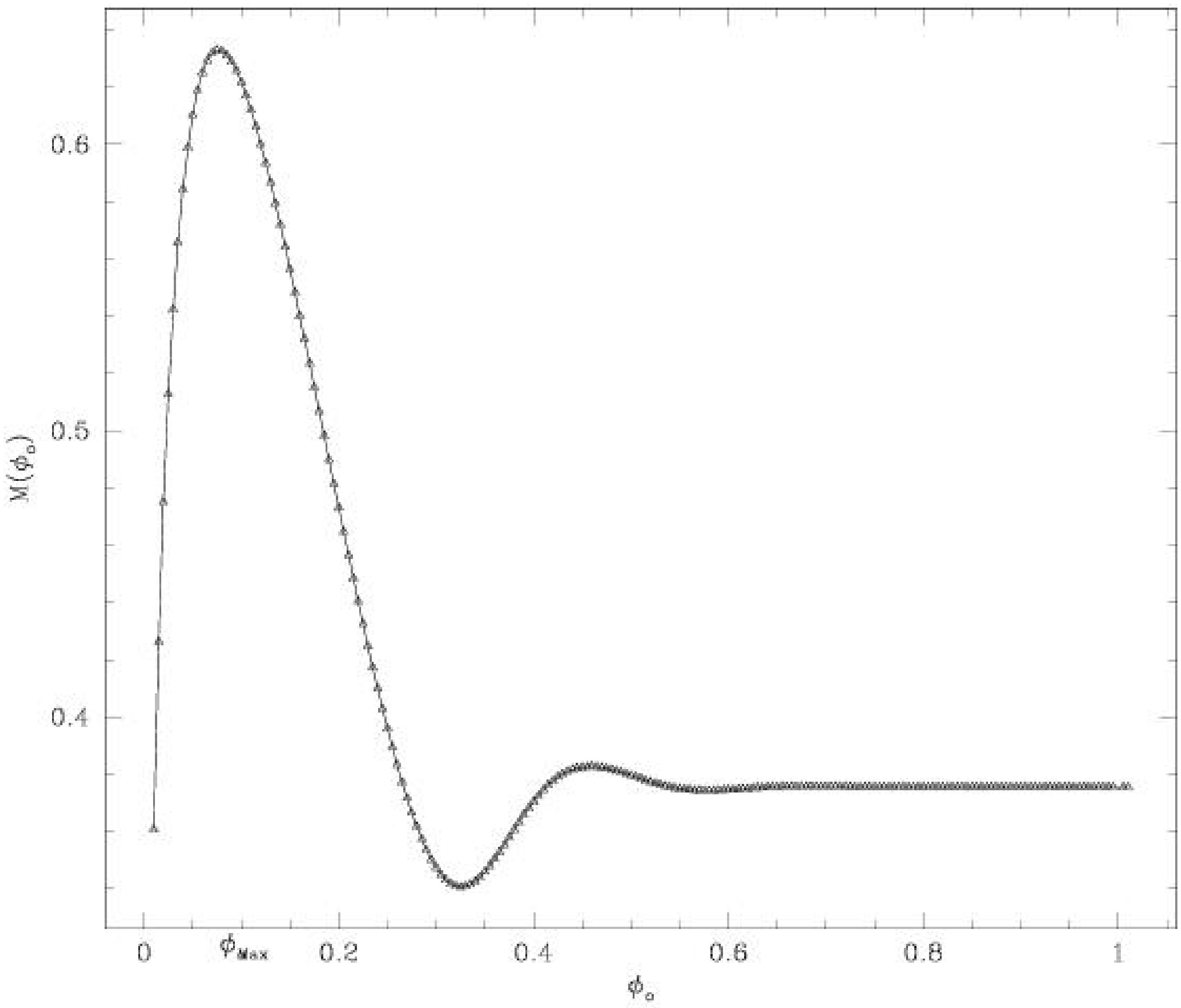
- Where for one specific value of ϕ_o we select the maximal value of z to represent this distribution

- This is repeated for all values of ϕ_o up to and including the value ϕmax



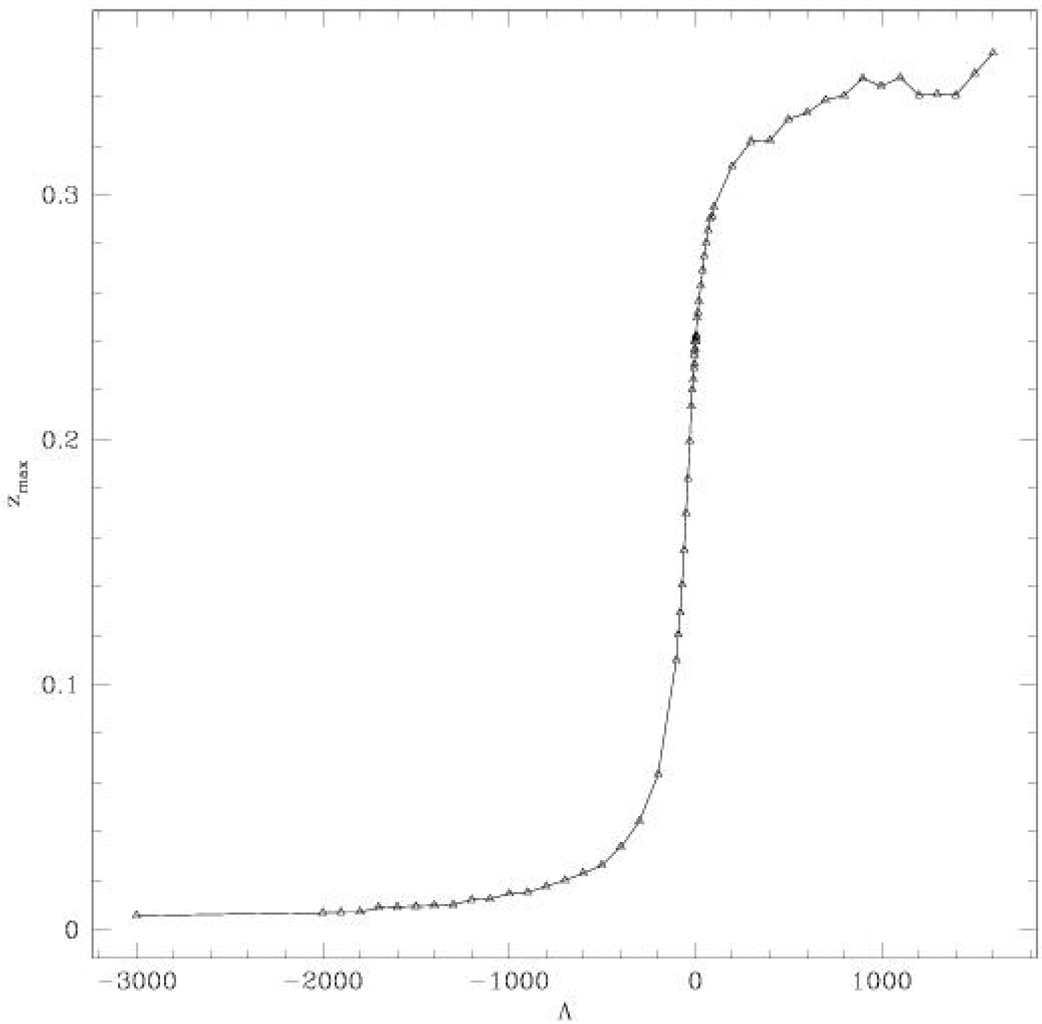
0.3
0.2
0.1
0
z
0
5
10
15
20
r

ϕ_{max}



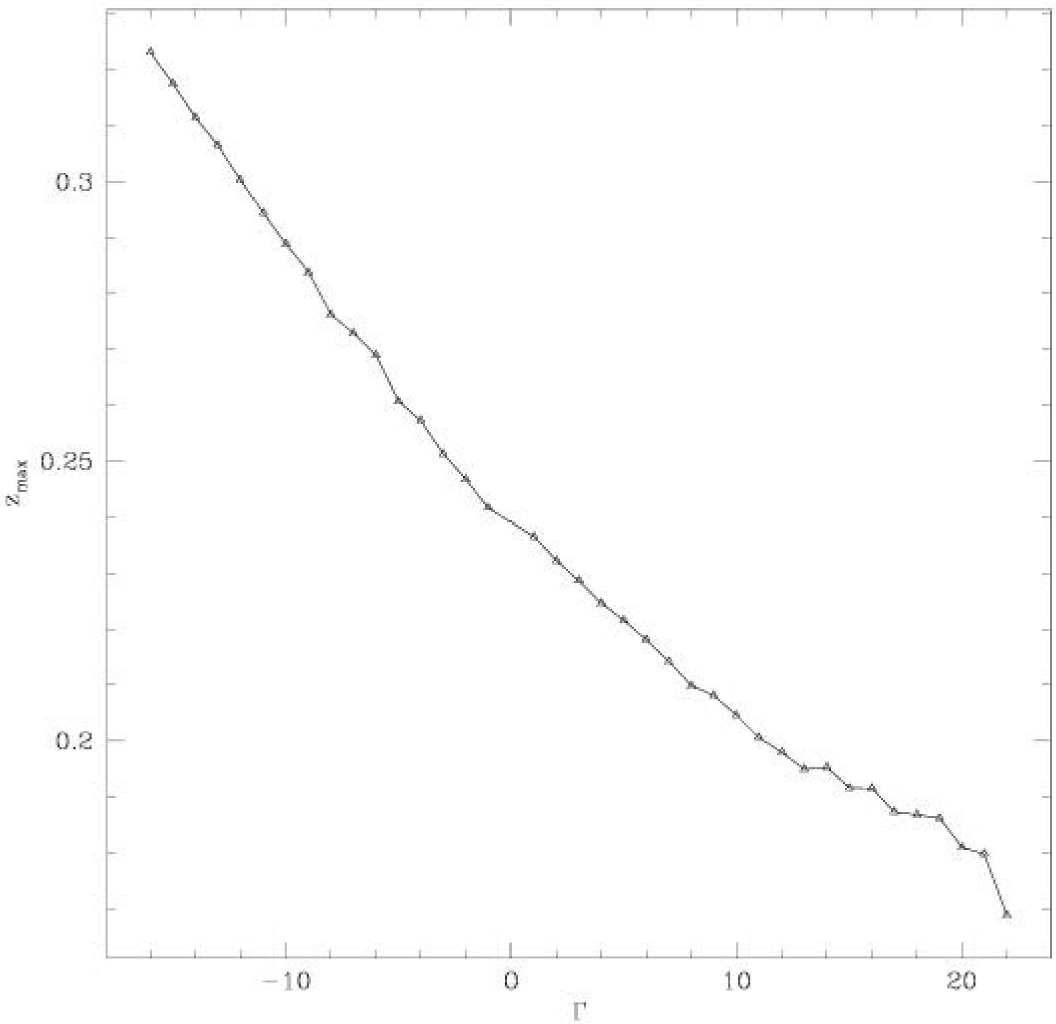
Collapse Function

- Gathering all the maximal z values for each phi, and again recording only the largest z for that particular value of the coefficient we are left with (for ϕ^4 self interaction);



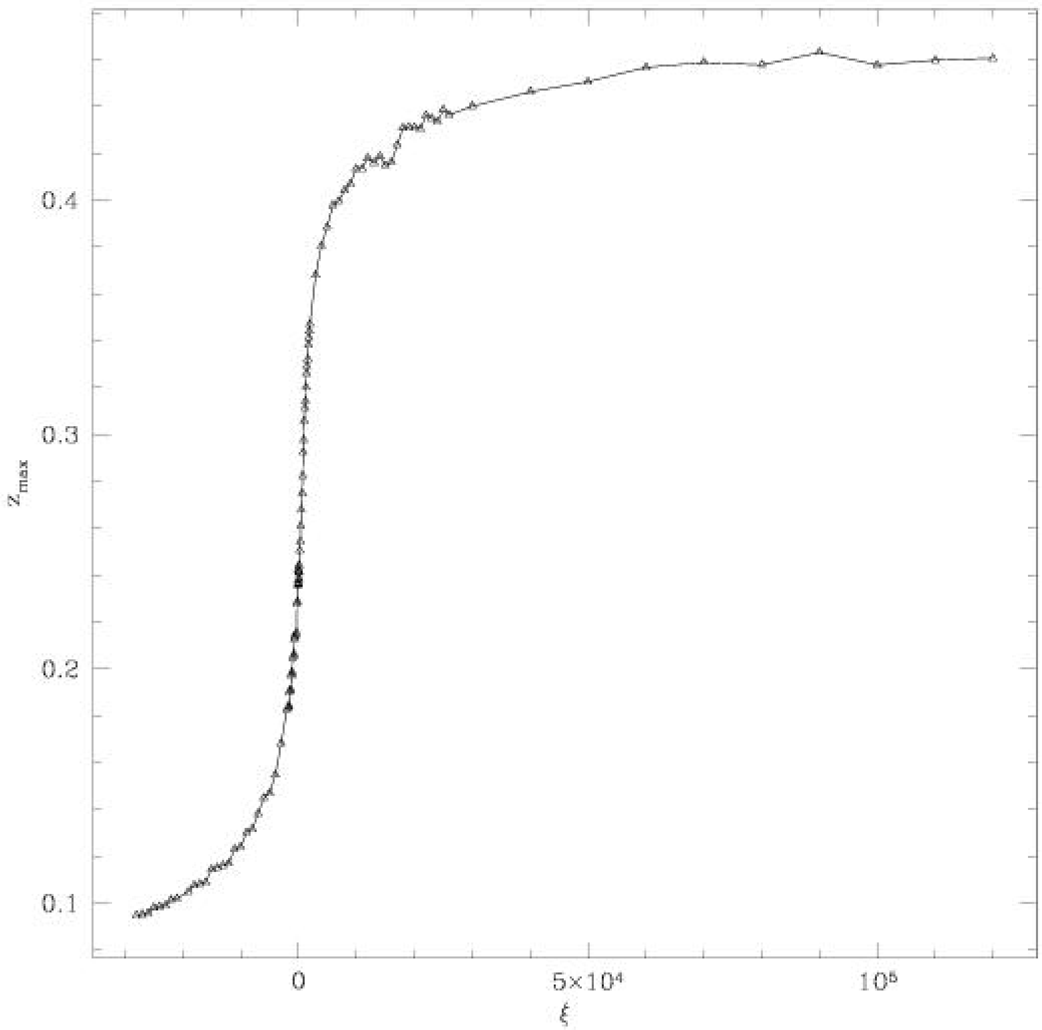
Collapse Function

- and for ϕ^3 self interaction

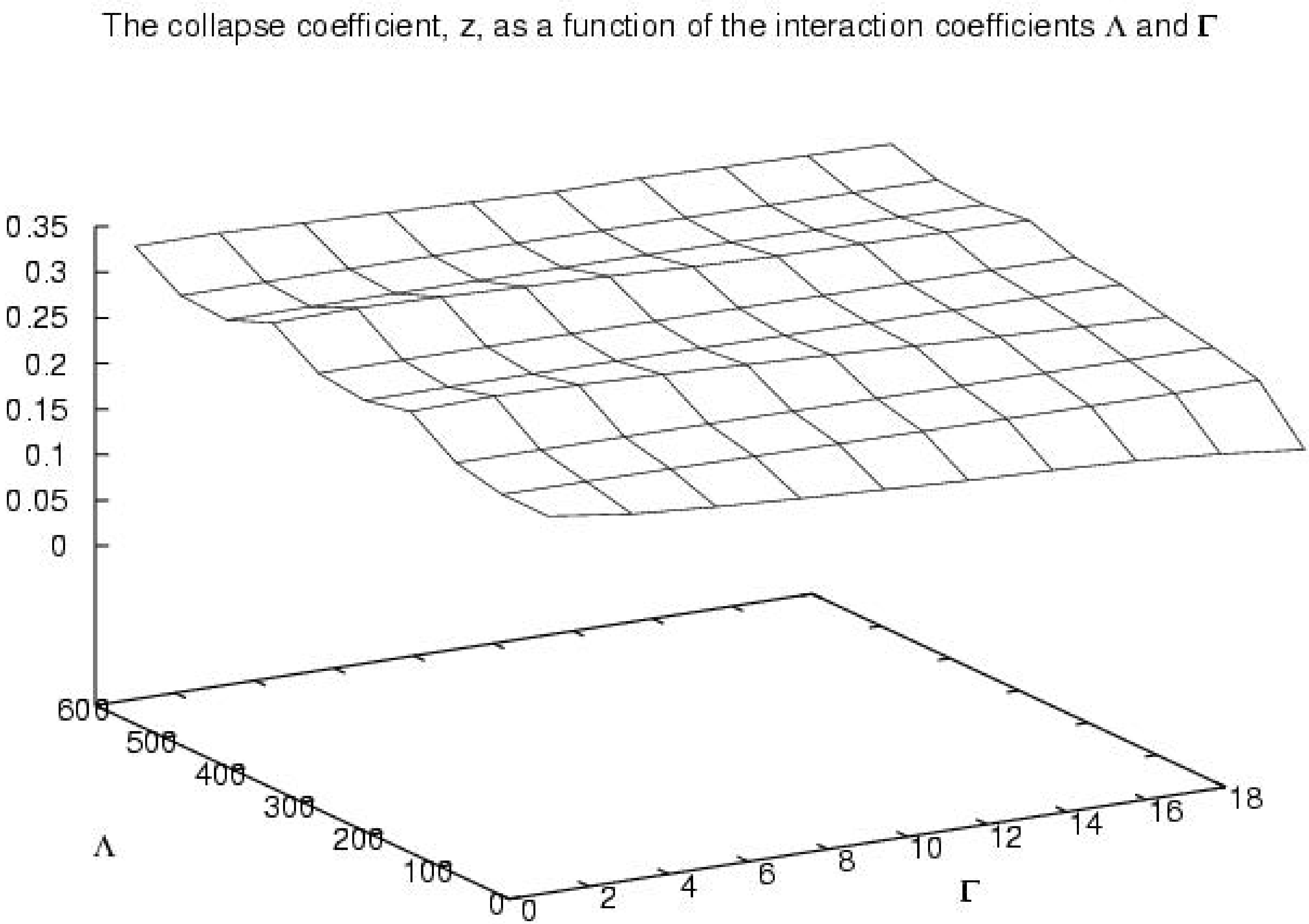


Collapse Function

- then for ϕ^6 self interaction



2 Parameter Survey - Preliminary results



Future Work

- A more detailed survey over the parameter space
- Possible extension to negative coupling constant in $\phi^3 + \phi^4$ parameter survey
- Extension to a $\phi^4 + \phi^6$ self interaction theory
- We could also apply this analysis to charged boson stars