

Ultra Relativistic Particle Collisions

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IX WORKSHOP NOVA FÍSICA NO ESPAÇO
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Work done in collaboration with Frans Pretorius
Dept of Physics, Princeton University

arXiv:0908.1780 (to appear in Phys Rev Lett)

Summary

- The Question

Do particle collisions of sufficiently high energy lead to black hole formation in general relativity?

- The Answer

You'll have to wait a few minutes!

- The Details

Results are basically as expected, but there are open questions

The Question

Do particle collisions of sufficiently high energy lead to black hole formation in general relativity?

Why is the question of current interest?

LHC: Large Hadron Collider



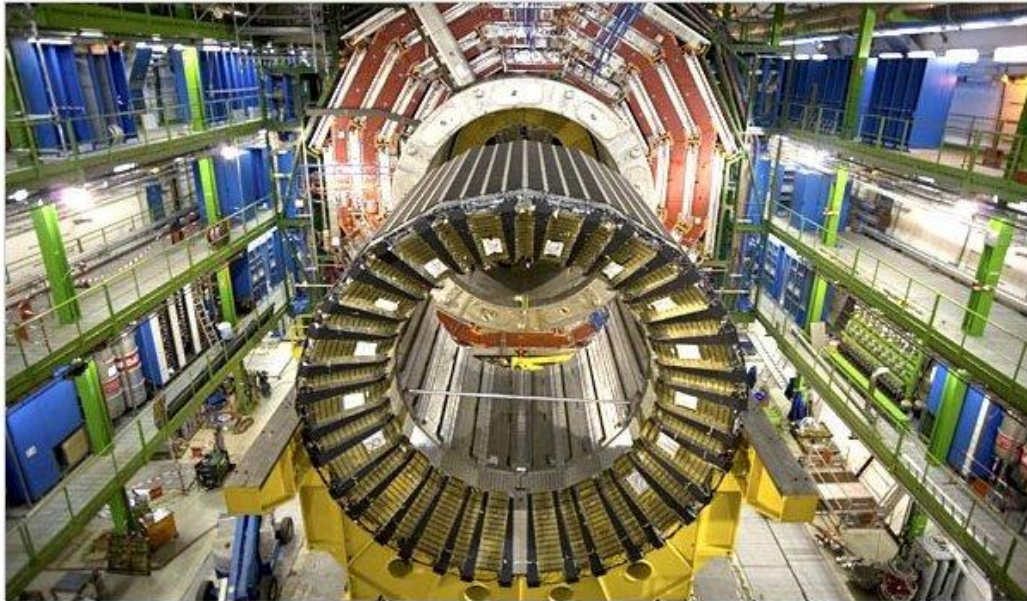
- World's largest/highest energy particle accelerator, currently operational at CERN, 27 km circumference tunnel spans French-Swiss border
- Will collide protons (hadrons) at total energies of about 15 TeV

SCIENCE

- Higgs boson (last key element of "Standard Model" which is unverified experimentally)
- Beyond the Standard Model
 - Supersymmetry?
 - Matter/anti-matter symmetry violations?
 - Large extra dimensions and ...

Black Hole Formation??

Asking a Judge to Save the World, and Maybe
a Whole Lot More



Part of a detector to study results of proton collisions by
lawsuit filed in Hawaii seeks to stop.

NYT March 29, 2008

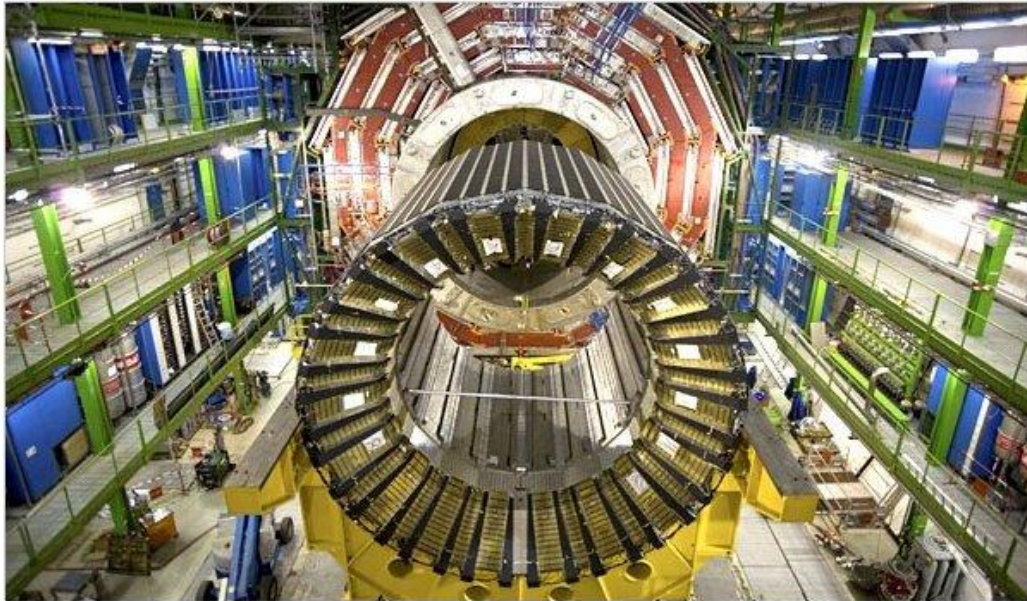
By DENNIS OVERBYE

Published: March 29, 2008

SIGN IN TO

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Walter L. Wagner and Luis Sancho contend that scientists at the European Center for Nuclear Research, or CERN, have played down the chances that **the collider could produce, among other horrors, a tiny black hole,** which, they say, could eat the Earth.

Physicists in and out of CERN say a variety of studies, including an official CERN report in 2003, have concluded there is no problem. But just to be sure, last year the anonymous Safety Assessment Group was set up to do the review again.

"The possibility that a black hole eats up the Earth is too serious a threat to leave it as a matter of argument among crackpots," said Michelangelo Mangano, a CERN theorist who said he was part of the group. The others prefer to remain anonymous, Mr. Mangano said, for various reasons. Their report was due in January.

“Fermi Calculation” for Planck Scale BH Formation

- From quantum mechanics (quantum field theory) and general relativity, **particle** of mass M defines two length scales

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 - de Broglie / Compton wavelength

$$L_c = \frac{hc}{E} = \frac{hc}{Mc^2} = \frac{h}{c} \frac{1}{M}$$

“Fermi Calculation” for Planck Scale BH Formation

- From quantum mechanics (quantum field theory) and general relativity, **particle** of mass M defines two length scales
 - de Broglie / Compton wavelength

$$L_c = \frac{hc}{E} = \frac{hc}{Mc^2} = \frac{h}{c} \frac{1}{M}$$

- Schwarzschild radius

$$L_s = \frac{2G}{c^2} M$$

“Fermi Calculation” for Planck Scale BH Formation

- Equate two length scales

$$L_c = L_s \rightarrow \frac{h}{c} \frac{1}{M} = \frac{2G}{c^2} M \rightarrow M^2 = \frac{hc}{G} \rightarrow M = \left(\frac{hc}{G} \right)^{1/2} \text{ (Planck mass)}$$

$$E = Mc^2 = \left(\frac{hc^5}{G} \right)^{1/2} \text{ (Planck energy)}$$

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For collision energies above Planck energy,
black hole formation is plausible

So ...

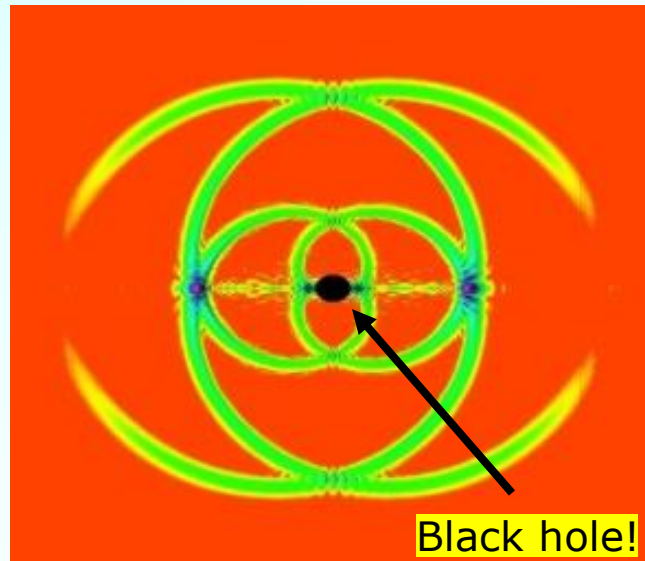
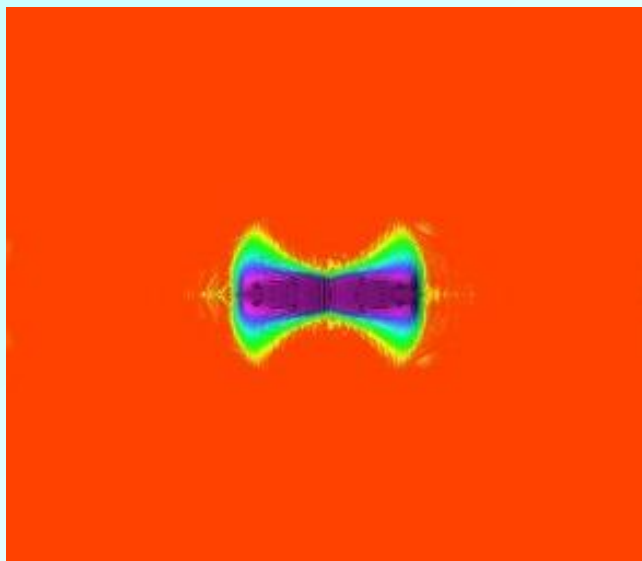
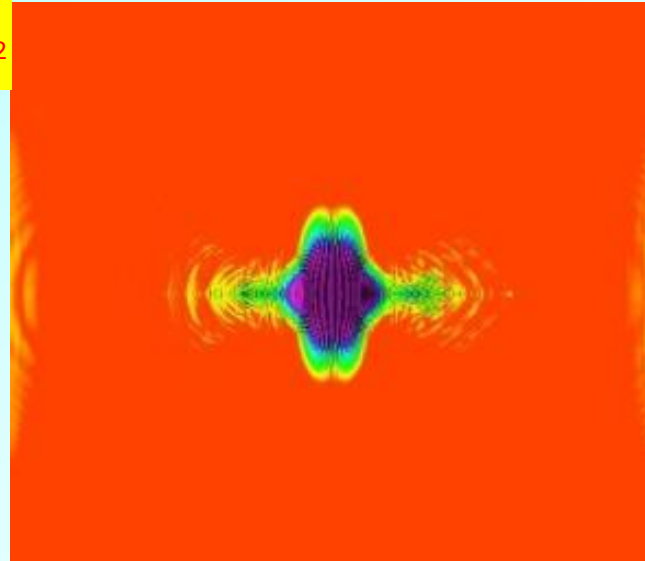
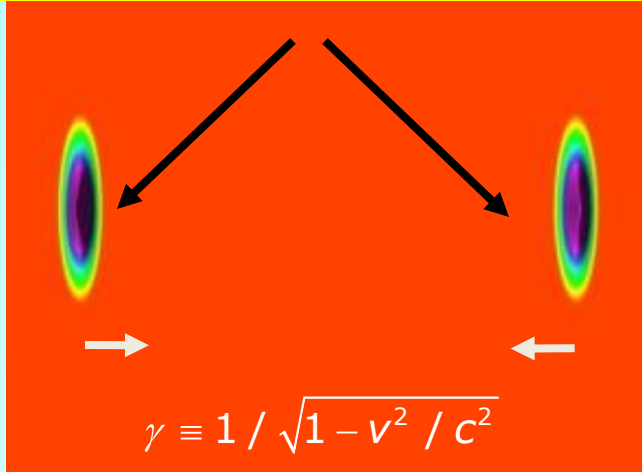
Do particle collisions of super-Planck energy lead to black hole formation in general relativity?

The Answer...

Yes!

Two identical particles

Rest mass m , Lorentz boost γ , Energy γmc^2



Black hole!

The Details

Outline of Remainder of Talk

- Mini-black-hole production via particle collisions
 - Review of standard chain of reasoning
- Possible weak links in the reasoning chain
 - Do (classical) collisions of particles at sufficiently high energy necessarily form black holes?
- Non-singular classical models for particles
 - Boson stars
- Results
- Open Questions
 - Nature of critical (threshold) solution

Gravitational Collapse – Black Hole Formation

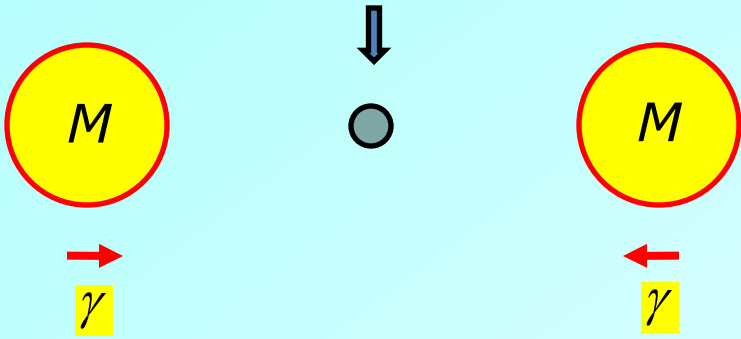
- Thorne (“Magic without Magic”, 1972): Hoop Conjecture

Horizons form when and only when a mass M gets compacted into a region whose circumference in *every* direction is

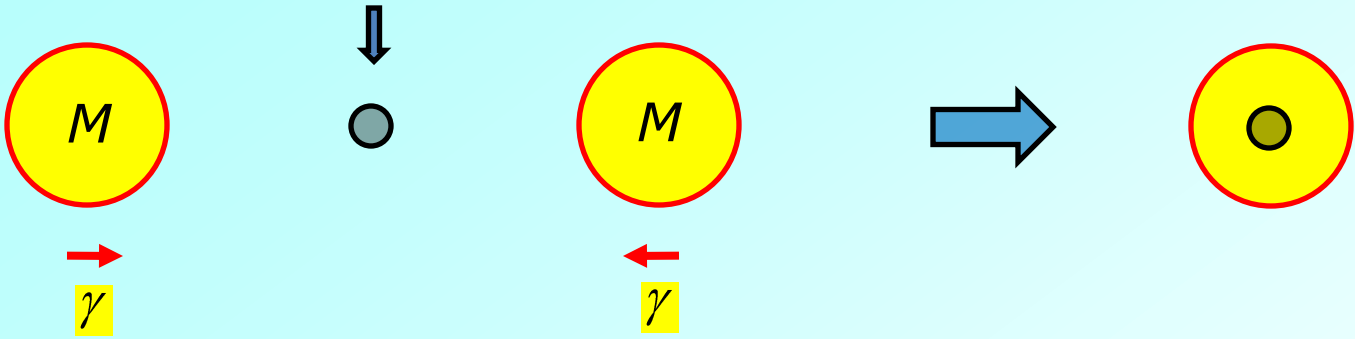
$$C \leq 2\pi R_S = \frac{4\pi G}{c^2} M$$

- Thorne: conjecture seems “eminently reasonable”, *but* is it useful/meaningful when spacetime is highly dynamical and non-linear?
- Domain of applicability not clear: Consider, e.g., single particle boosted beyond Planck energy: No gravitational collapse

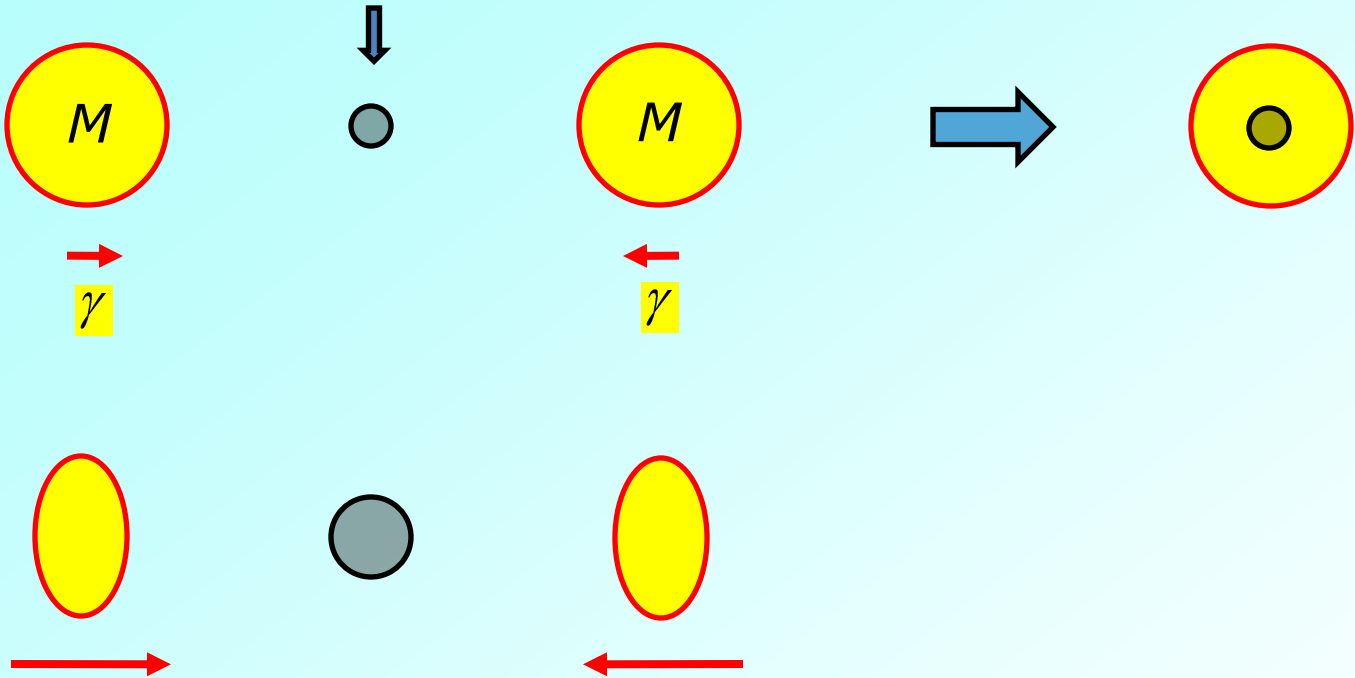
hoop radius: $2 \times 2\gamma M$



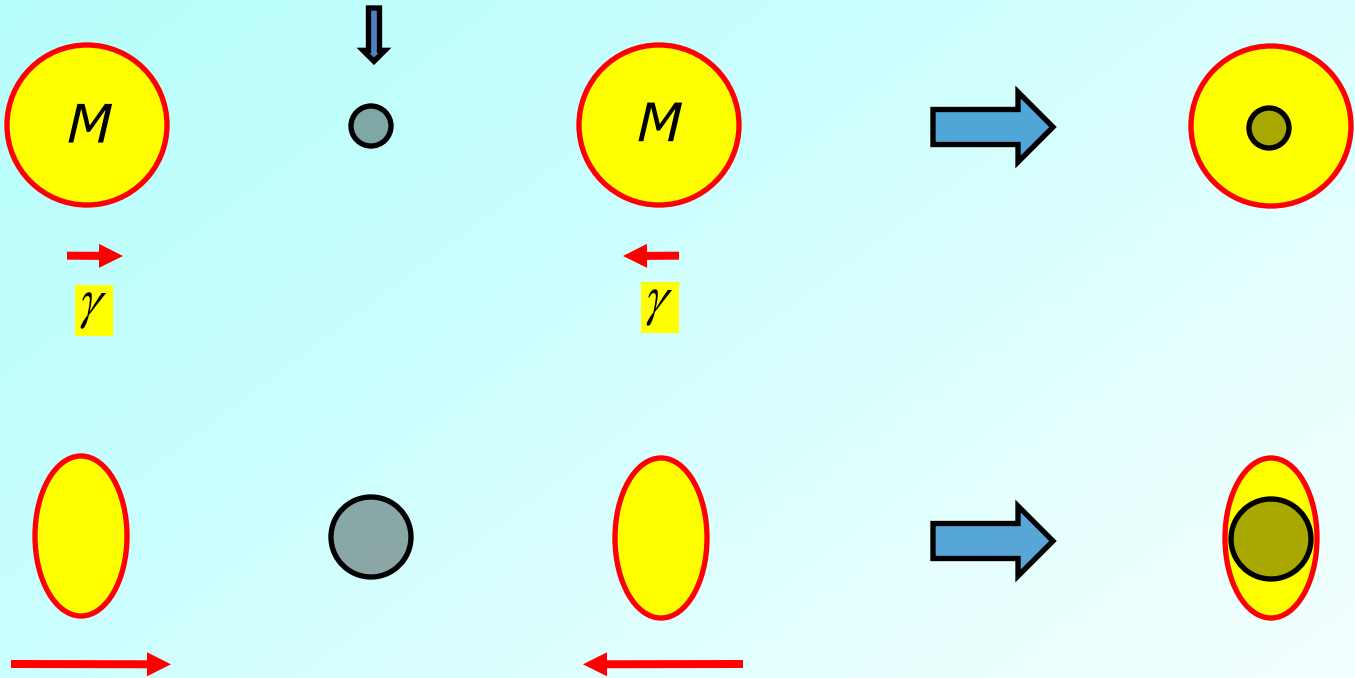
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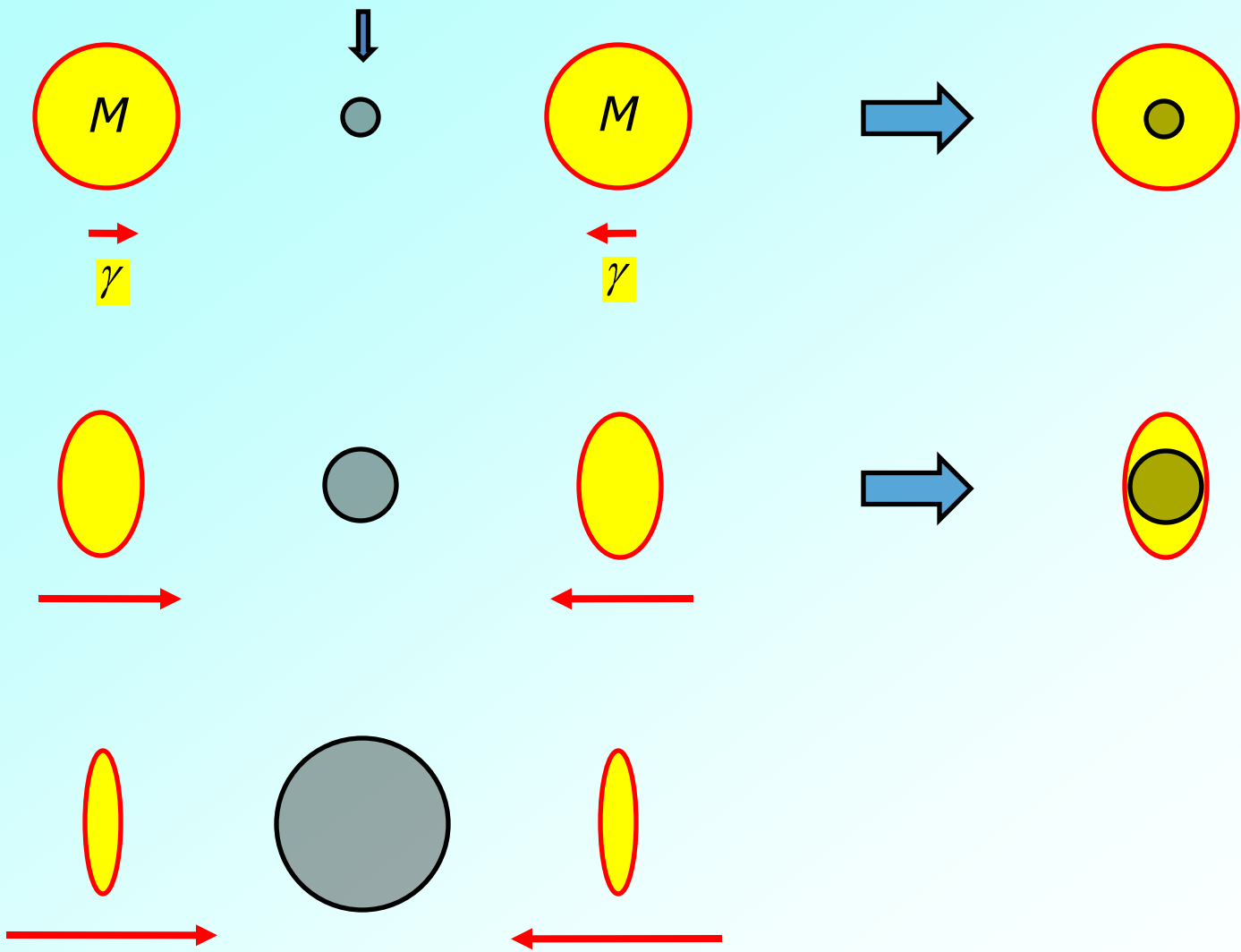
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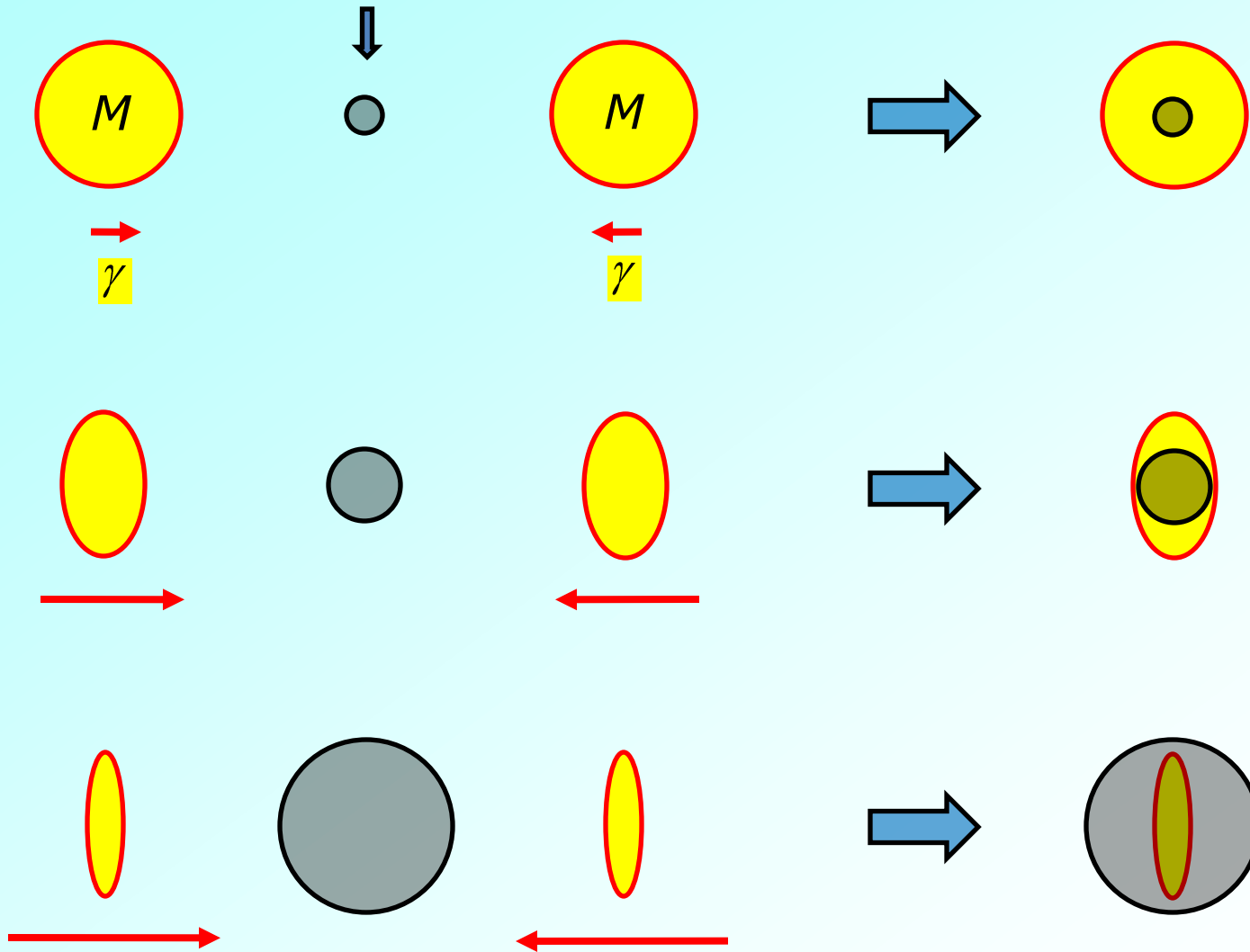
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black hole formation plausible

- Will now use “natural” units

$$G = c = \hbar = 1$$

- Very few specific references in discussion of standard story for black hole production, see review articles such as
 - “High –energy black hole production”, Giddings, arXiv:0709.1107
 - “Black holes at the LHC”, Kanti, arXiv:0802.2218

(A) Standard scenario for black hole formation in accelerators such as the LHC

- Universe needs more spatial dimensions – D total, $D - 1$ spatial - than the 3 we routinely experience
 - String theories require additional dimensions for mathematical & physical consistency (typically, $D - 1 = 10$)
 - Assume some string theory model & assume extra dimensions compact, and small, but not necessarily of the order of the 4-dimensional Planck length (10^{-33}cm , 10^{19} GeV)
- **Basic idea:** Black holes will be able to form when collision energies of particles exceed the “real” Planck mass, M_D
- Need mechanism to get M_D to low energies, TeV scale if we are to see black hole production at LHC

Standard Scenario (cont.)

- Such mechanisms/scenarios have existed since the early '90s
 - Large extra dimensions (Arkani-Hamed, Dimopoulos, Dvali; Antoniadis et al, ...)
 - Large warping with brane-world scenario (Randall & Sundrum, ...)
- Example: Large warping

$$ds^2 = e^{2A(y)} dx_4^2 + g_{mn}(y) dy^m dy^n \quad m, n = 5 \dots D$$

- Relationship between 4-d and D -d Planck masses

$$M_4^2 = M_D^{D-2} \int \frac{d^{D-4} y}{(2\pi)^{D-4}} \sqrt{g_{D-4}} e^{2A}$$

If “warped” volume large enough, can have $M_D \ll 10^{19}$ GeV

Standard Scenario (cont.)

- For collision energies $E \gg M_D$, assume that quantum gravity effects are ignorable – suppression of effects by powers of M_D / E
- Thus assume that we can describe very high energy collision of particles as a classical process, characterized by relative boost parameter, $\gamma \equiv 1 / \sqrt{1 - v^2}$ and an impact parameter, b
- Further assume that $\gamma \gg 1$ case is well approximated by $\gamma \rightarrow \infty$ or perturbations thereof
- Assume that spacetime of “infinitely boosted” particle is given by spacetime of “infinitely boosted” black hole (kinetic energy dominance, “massless particle”)

Standard Scenario (cont.)

- Aichelburg & Sexl (GRG, 1971) – applied boost to Schwarzschild metric, taking limit $\gamma \rightarrow \infty$ while keeping lab-frame energy, p , fixed
- Find

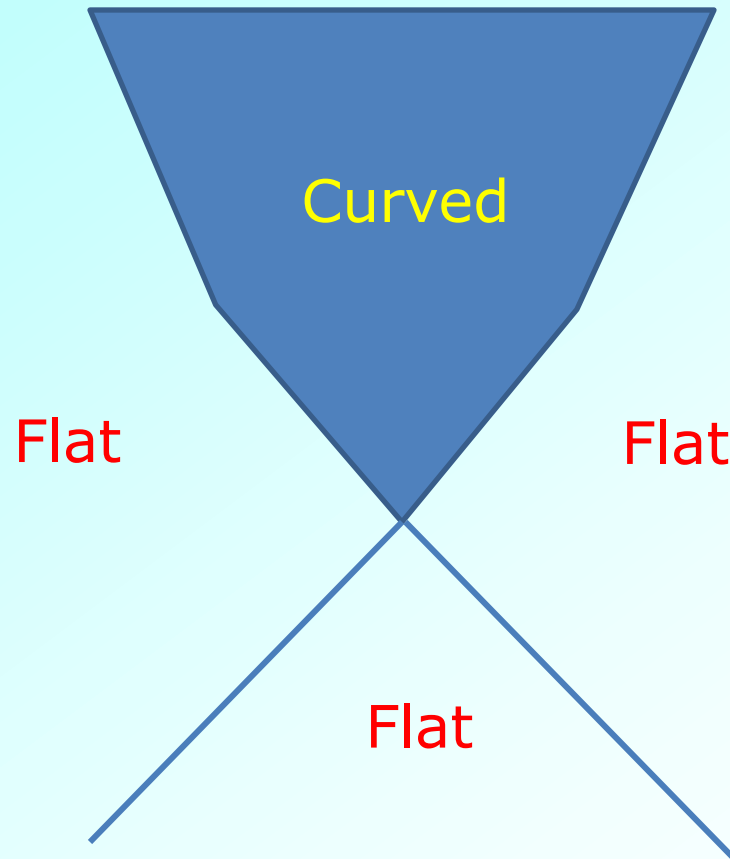
$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2 + 4p \left\{ \frac{1}{|t-x|} - 2\delta(t^2 - x^2) \ln(y^2 + z^2)^{1/2} \right\} (dt - dx)^2$$

- Can be interpreted as a plane-fronted gravitational “shock wave” (type of PP-wave), spacetime flat except at $t = x$, where certain components of curvature tensor have δ -function singularities

Standard Scenario (cont.)

- Collisions of two infinitely boosted black holes: can “glue” two Aichelburg-Sexl solutions to get geometry everywhere outside future light cone of collision event
- Geometry analyzed by Penrose in early 70’s for head on collisions, found apparent horizons (compact spacelike 2-surfaces with vanishing divergence of outgoing null geodesics) on “shock surfaces”
- Assuming cosmic censorship, trapped surface implies black hole, so (larger) black hole will form even for non-zero impact parameter

Schematic Spacetime Diagram of Collision



Standard Scenario (cont.)

- Penrose calculations (location of trapped surfaces in shock wave geometries) extended to D -dimensional case by Eardley & Giddings (PRD, 66, 044011 (2002)) yielding improved estimates of cross sections for black hole formation (relative to naïve estimates based on scaling arguments)
- Eardley & Giddings calculations have been further extended, and much of the post-collision / black hole formation physics has been studied extensively
 - Decay of black holes particularly important, especially vis a vis signatures for experimental detection various phases of decay identified

Standard Scenario (cont.)

FOR PURPOSES OF THIS TALK – THAT'S ALL!!

Possible difficulties with standard scenario

- Development relies rather crucially on use of Aichelburg-Sexl (AS) solutions to model ultra-high-energy particles coupled to the gravitational field
 - AS metric is not asymptotically flat
 - Algebraic type of metric changes from Petrov Type D (two distinct null eigenvectors of Weyl) to Petrov type N (one null eigenvector)
 - Metric does not provide good description of finite boosted particle on the shock surface
- These concerns plus indications from earlier calculations suggesting that gravity might actually “weaken” for high-energy collisions in part motivated our “direct assault” on the problem

Strategy

- Assume that calculations in 3+1 dimensions can shed light on $D+1$ dimensions (analogously to use of higher-dimensional Aichelburg-Sexl solutions in standard scenario)
 - Investigate collision dynamics using non-singular (i.e. non point like, non-black-hole) models for particles that are coupled to the gravitational field
 - Start with head-on collisions, view black hole formation in ultra relativistic limit of such collisions as **necessary** condition for black hole formation at finite impact parameter
 - Use simplest matter models available, with understanding that should ultimately use as wide range of such models (interactions etc.) as possible

Non-singular classical models for particles: Boson Stars

- Want to avoid point-like description of “particle”
- Look for regular (non-singular), compact configurations of classical fields with time-independent stress-tensors
- Simple case: Single complex field with self-interaction potential
- Lagrangian density

$$-\frac{1}{2} \phi_{,\mu} \phi^{*,\mu} - V(\phi)$$

Boson Stars

- Coupled Einstein-Klein-Gordon equations

$$\begin{aligned}G_{\mu\nu} &= 8\pi T_{\mu\nu} \\T_{\mu\nu} &= \frac{1}{2}(\phi_{,\mu}\phi_{,\nu}^* + \phi_{,\mu}^*\phi_{,\nu}) - \frac{1}{2}g_{\mu\nu}\phi_{,\alpha}\phi^{*,\alpha} - g_{\mu\nu}V \\ \phi_{,\mu}{}^{i\mu} &= \frac{dV}{d\phi}\end{aligned}$$

- Look for static, spherically symmetric solutions (Kaup, PR 172, 1331 (1968), Ruffini & Bonazzola, PR 187, 1767 (1969)...)
- Choose “Schwarzschild-like” coordinates

$$ds^2 = -\alpha(r)^2 dt^2 + a(r)^2 dr^2 + r^2 d\Omega^2$$

and adopt ansatz

$$\phi(t, r) = \Phi(r) e^{i\omega t}$$

$\omega \equiv$ eigenvalue of problem

(Mini) Boson Stars

- Configurations of sought type exist for broad range of potentials; simplest choice has only a mass term

$$V(\phi) = m^2 |\phi|^2 \equiv |\phi|^2$$

“Mini” nomenclature comes from fact that for any plausible particle mass-parameter, gravitating mass of typical boson star is tiny compared to typical fluid star

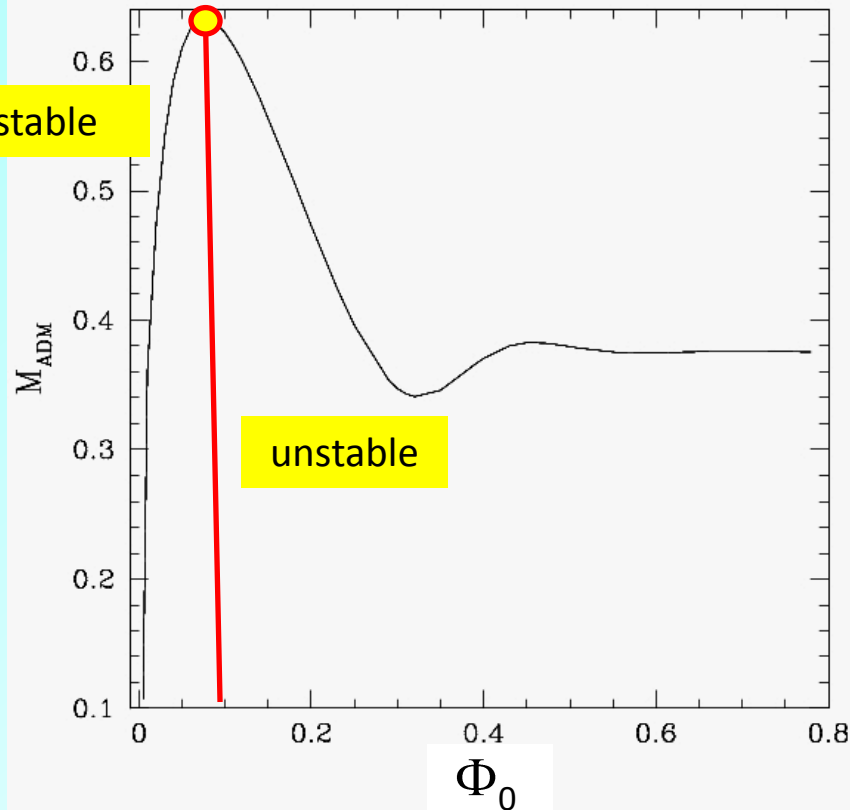
- Solutions comprise one-parameter family: conveniently labelled by central modulus

$$\Phi_0 \equiv \Phi(0) = |\phi(t, 0)|$$

$$\omega \equiv \omega(\Phi_0)$$

Mini Boson Stars

Gravitational Mass vs Central Field Modulus



- Typical of “relativistic stars”: for any given potential (equation of state), sequence exhibits maximum mass
- Stars to left/right of mass maximum are perturbatively stable/unstable respectively
- Each extremum in plot signals additional unstable mode

Newtonian Boson Stars

- Non-relativistic limit of above system – self gravitating Schrödinger equation

$$i \frac{\partial \phi}{\partial t} = -\nabla^2 \phi + V_g \phi$$
$$\nabla^2 V_g = 4\pi |\phi|^2$$

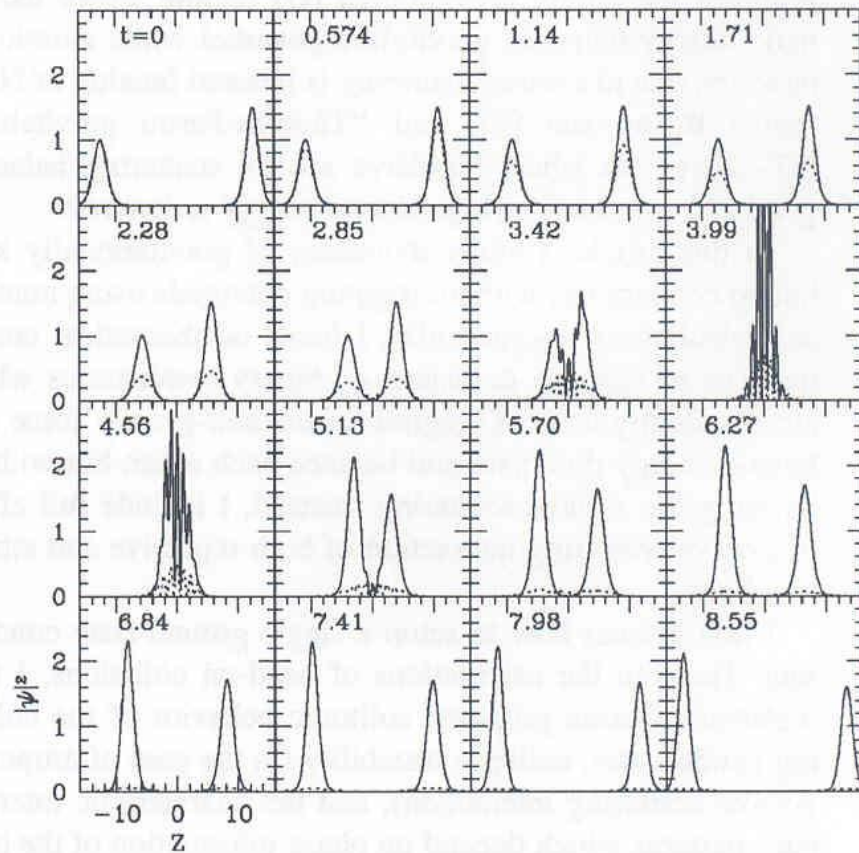
- Make same ansatz as previously, again find one-parameter family of solutions parametrized by central modulus of scalar field
- In this case, gravitating mass increases monotonically with and different solutions can be determined from one another via appropriate re-scalings.

Key Question

What happens when boson stars collide??

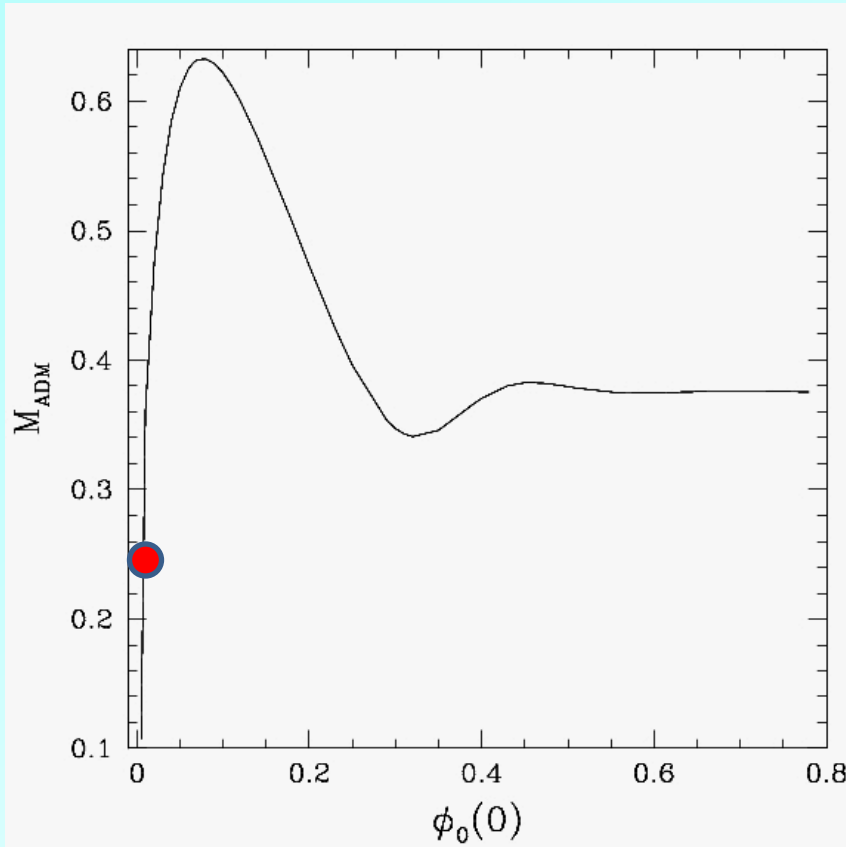
- Will look at three examples, all involving head on (axisymmetric) collisions of two boson stars which are boosted towards one another
 - Newtonian mini-boson stars (D. Choi, PRA, 66, 063609 (2002))
 - Relativistic boson stars I (K. Lai, UBC PhD thesis, (2004), includes quartic term in $V(\phi)$)
 - Relativistic boson stars II (MWC & Pretorius, PRL in press)

Head-on Collision of Newtonian Boson Stars



- Sequence shows evolution of $|\phi|^2$ along axis of symmetry for two stars with somewhat different masses.
- Stars pass through one another relatively unscathed; i.e. stars exhibit “solitonic” behaviour
- (Similar behaviour seen when gravitational self-interaction replaced by cubic term in Schrödinger equation)

Head-on Collision of Relativistic Boson Stars I



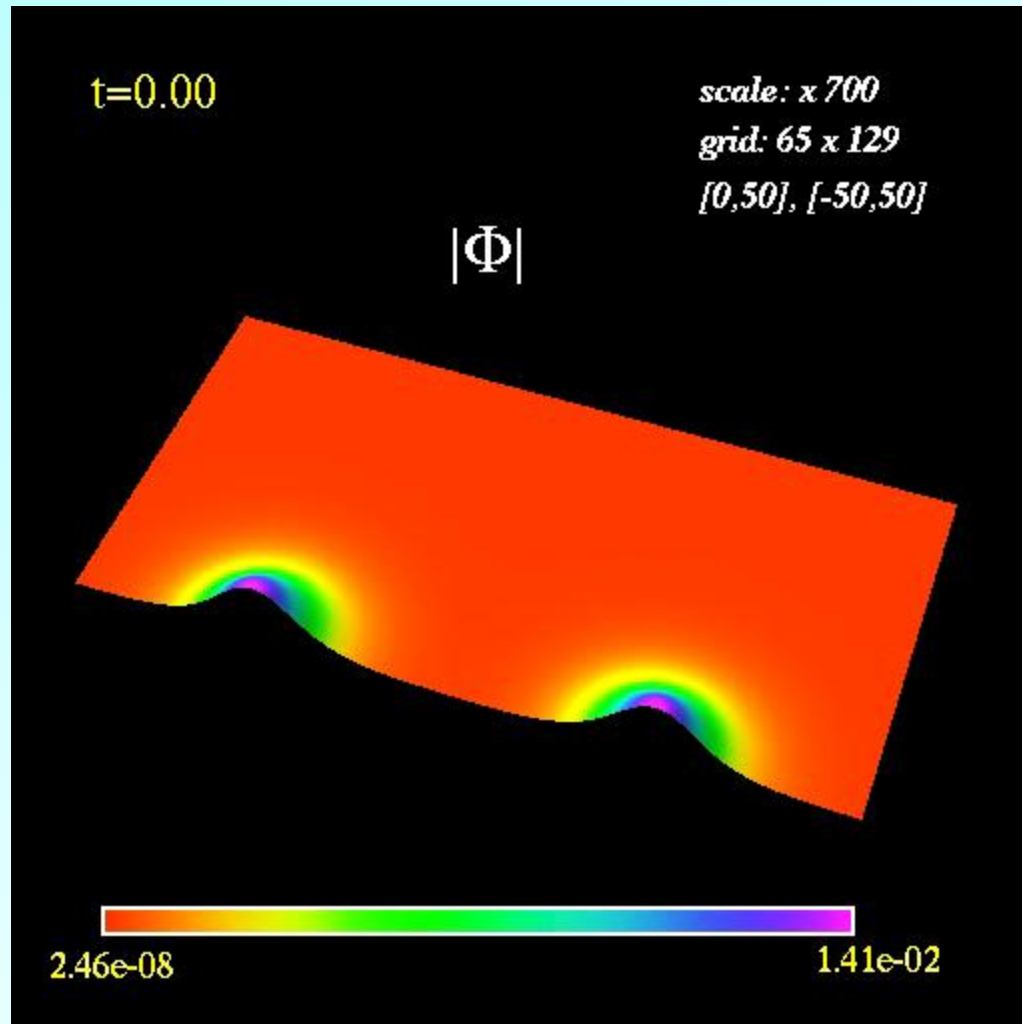
- Potential

$$V(\phi) = |\phi|^2 + \frac{1}{2} |\phi|^4$$

- Identical stars well separated from mass maximum are boosted towards one another
- Investigate behaviour as function of magnitude of boost

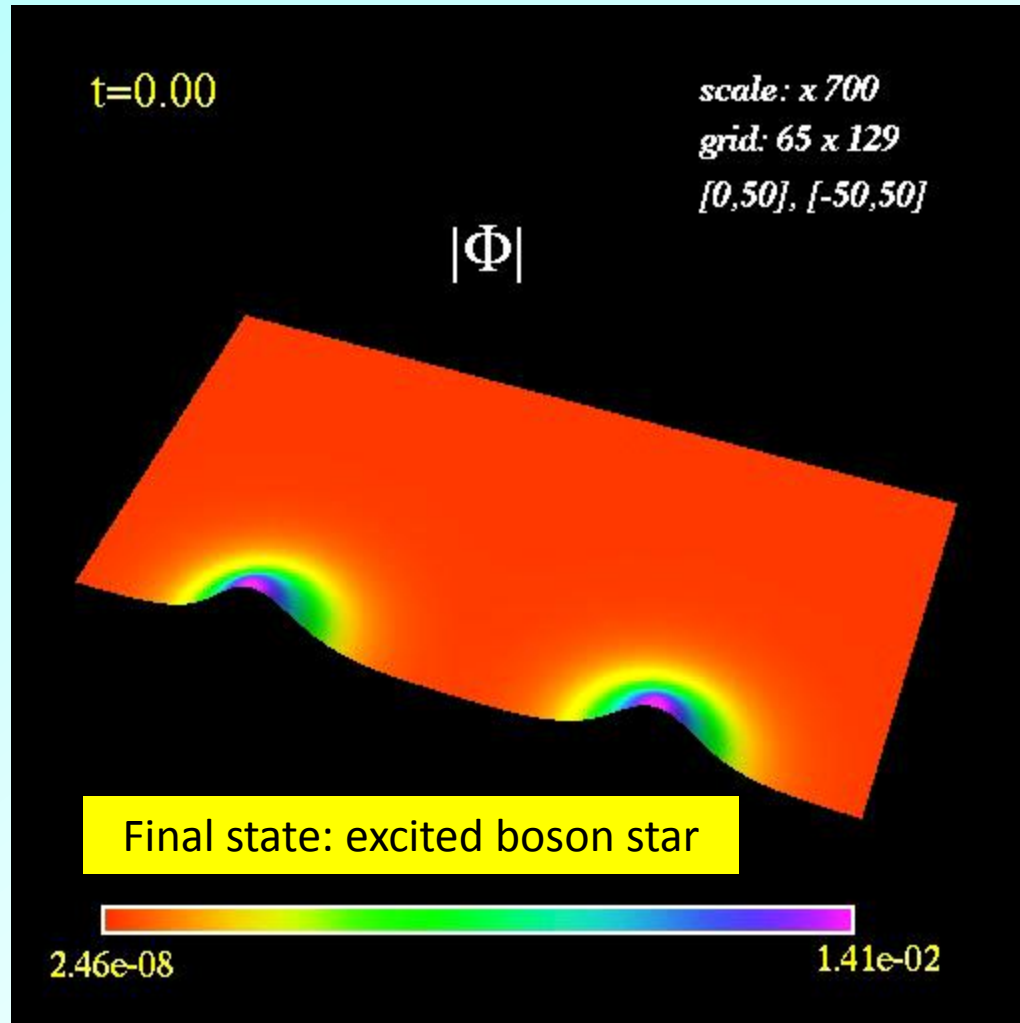
Head-on Collision of Relativistic Boson Stars I (stars start from rest)

$|\phi(t, \rho, z)|$



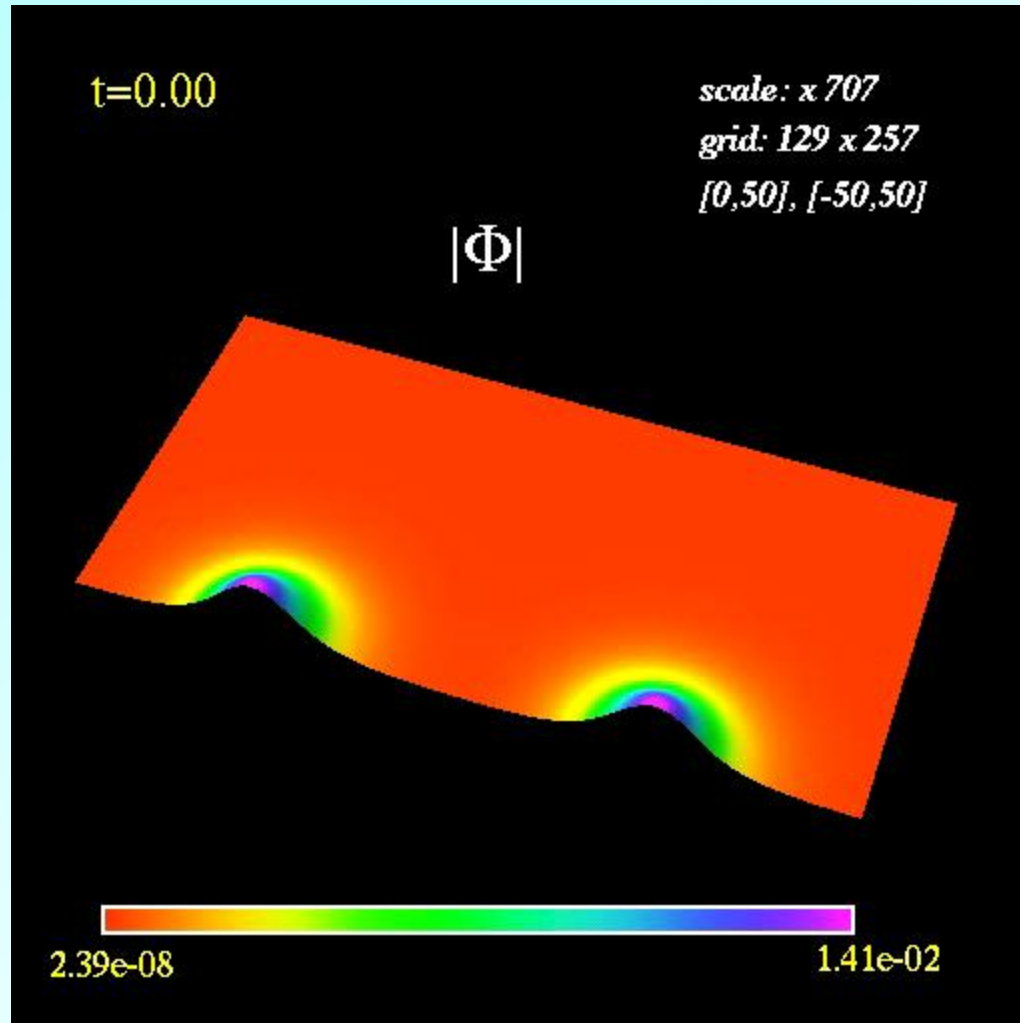
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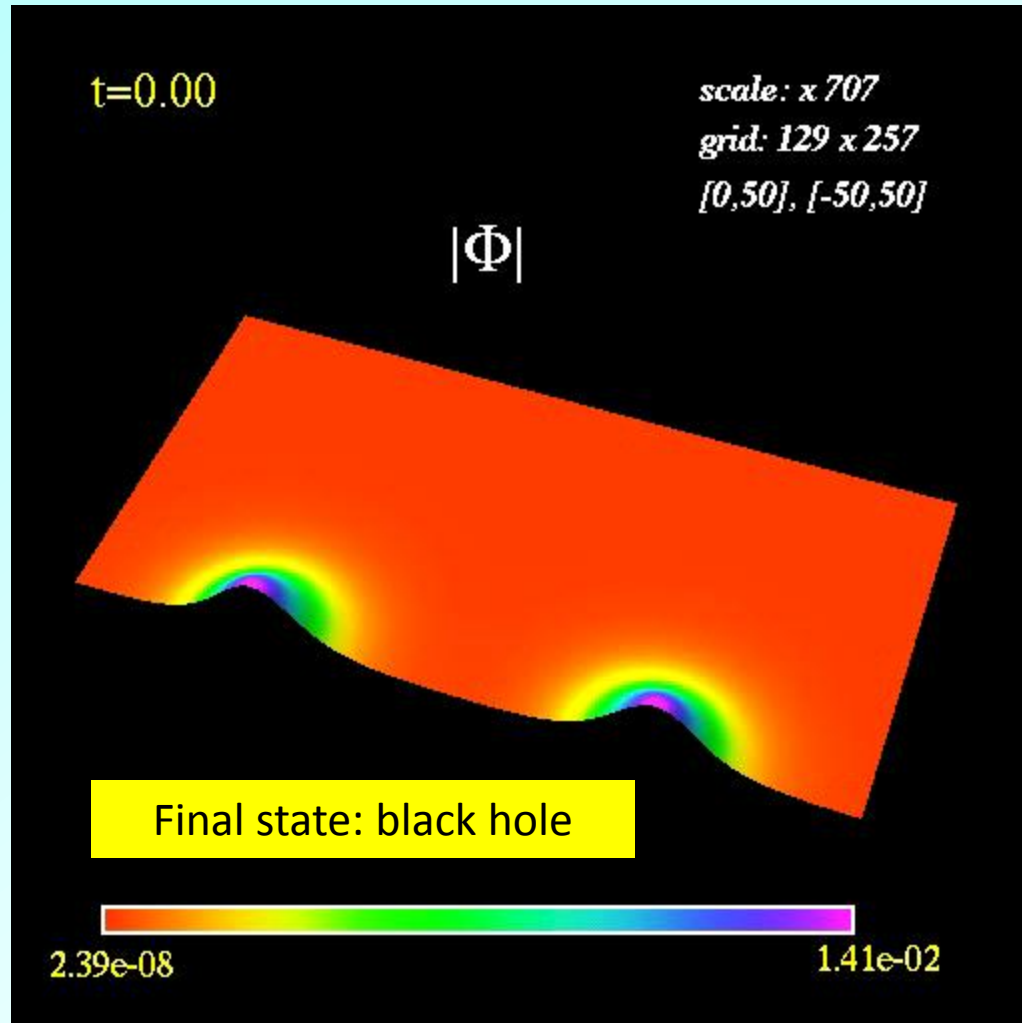
Head-on Collision of Relativistic Boson Stars I (small initial boost)

$|\phi(t, \rho, z)|$



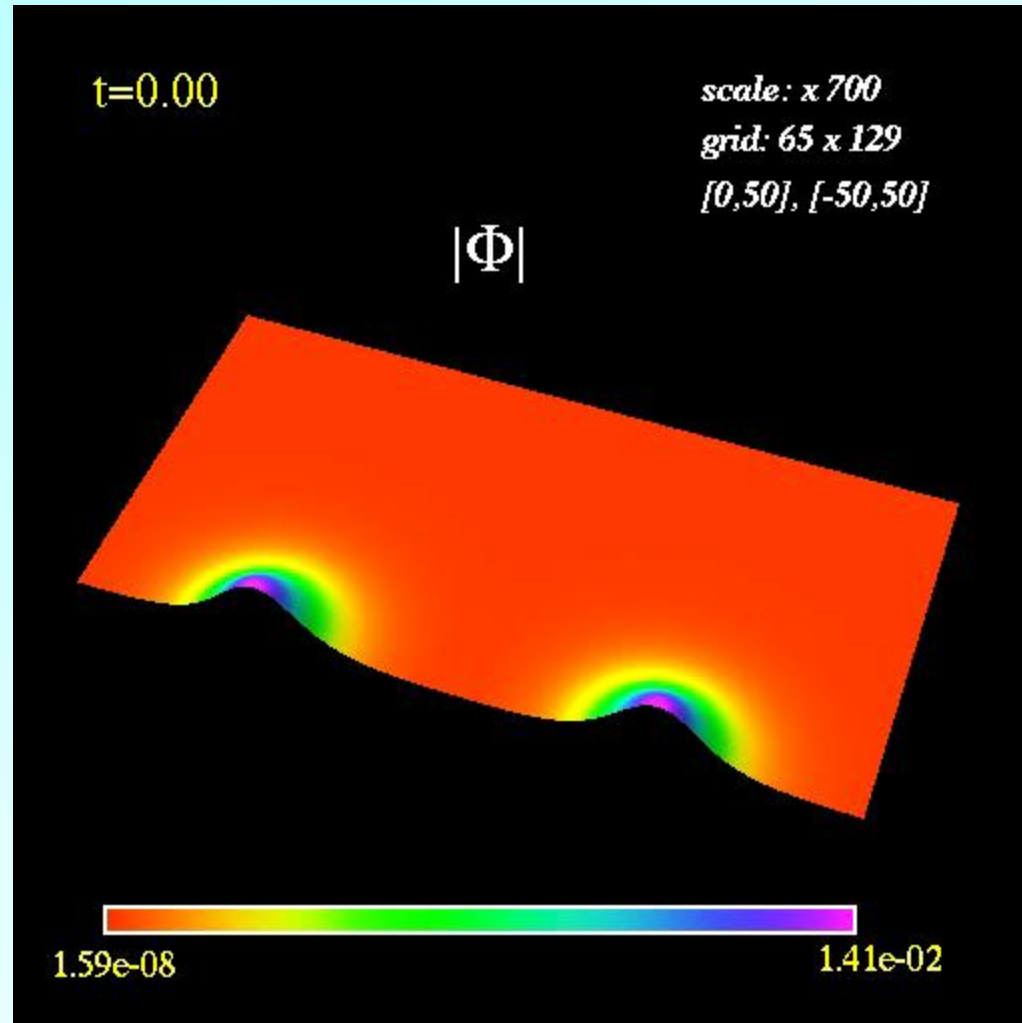
Head-on Collision of Relativistic Boson Stars I (small initial boost)

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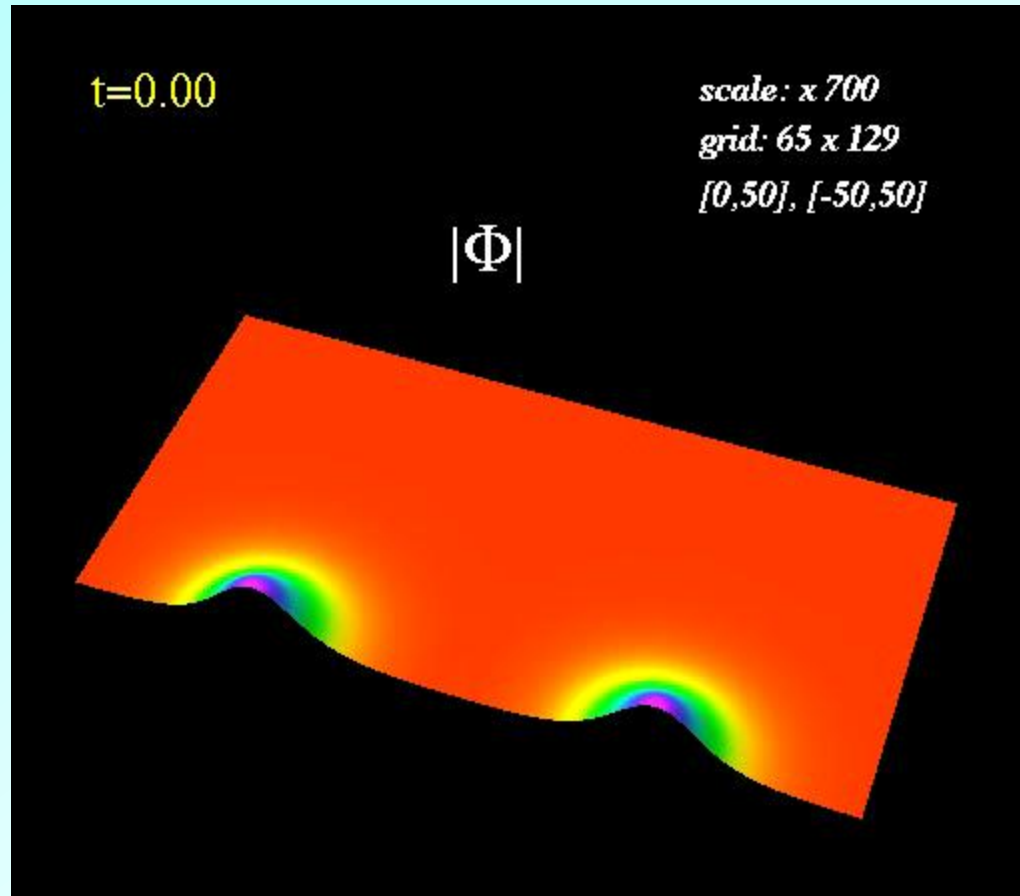
Head-on Collision of Relativistic Boson Stars I (large initial boost, $v \approx 0.7$)

$|\phi(t, \rho, z)|$



Head-on Collision of Relativistic Boson Stars I (large initial boost, $v \approx 0.7$)

$|\phi(t, \rho, z)|$



Stars pass through one another, but not yet in regime where hoop conjecture would suggest black hole should form

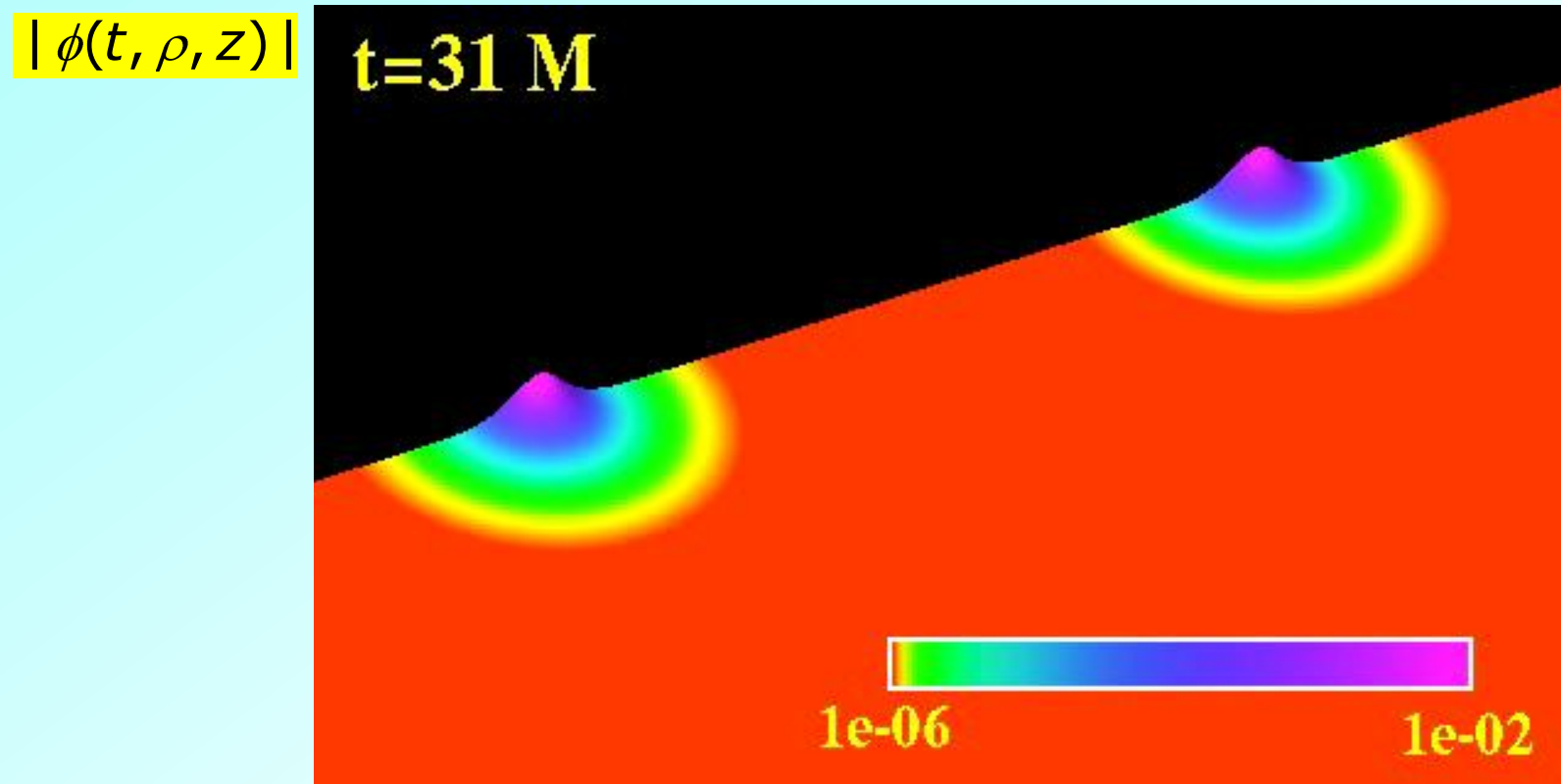
Head-on Collision of Relativistic Boson Stars II

(MWC & Pretorius, arXiv:0908.1780, to appear in PRL)

- Animations show 4 distinct calculations
 - Identical boson stars used in all cases, initial boost velocities (parameterized by $\gamma \equiv 1 / \sqrt{1 - v^2}$) vary
 - $\max(2m(r) / r) \approx 1 / 22$
 - Hoop Conjecture: Black hole formation when $\gamma \approx 11$
- Values of γ used: 1, 2, 4, 3.125

Head-on Collision of Relativistic Boson Stars II

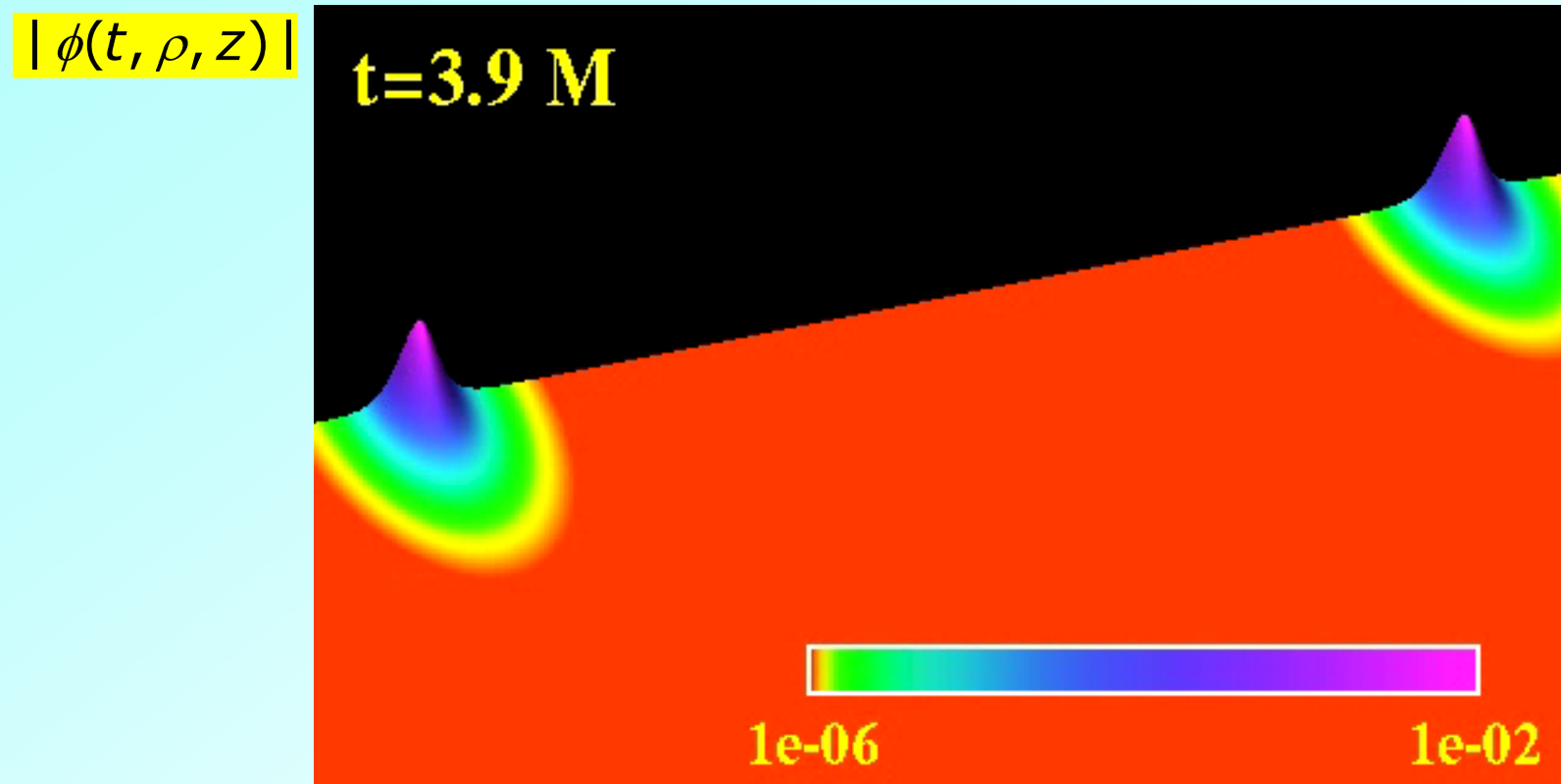
($\gamma = 1$)



Collision results in “perturbed” boson star (eventually collapses to BH)

Head-on Collision of Relativistic Boson Stars II

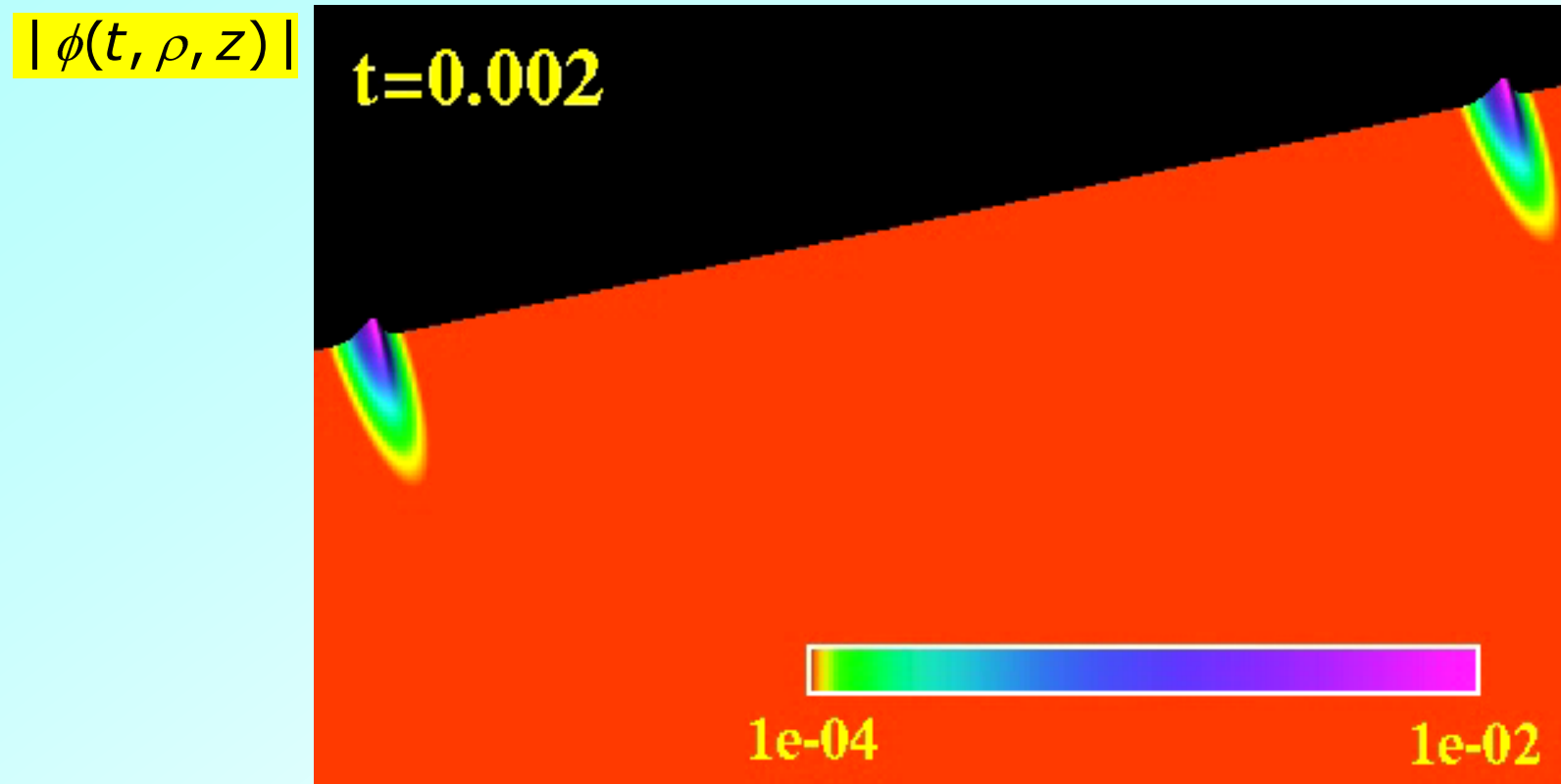
($\gamma = 2$)



Collision is "solitonic"

Head-on Collision of Relativistic Boson Stars II

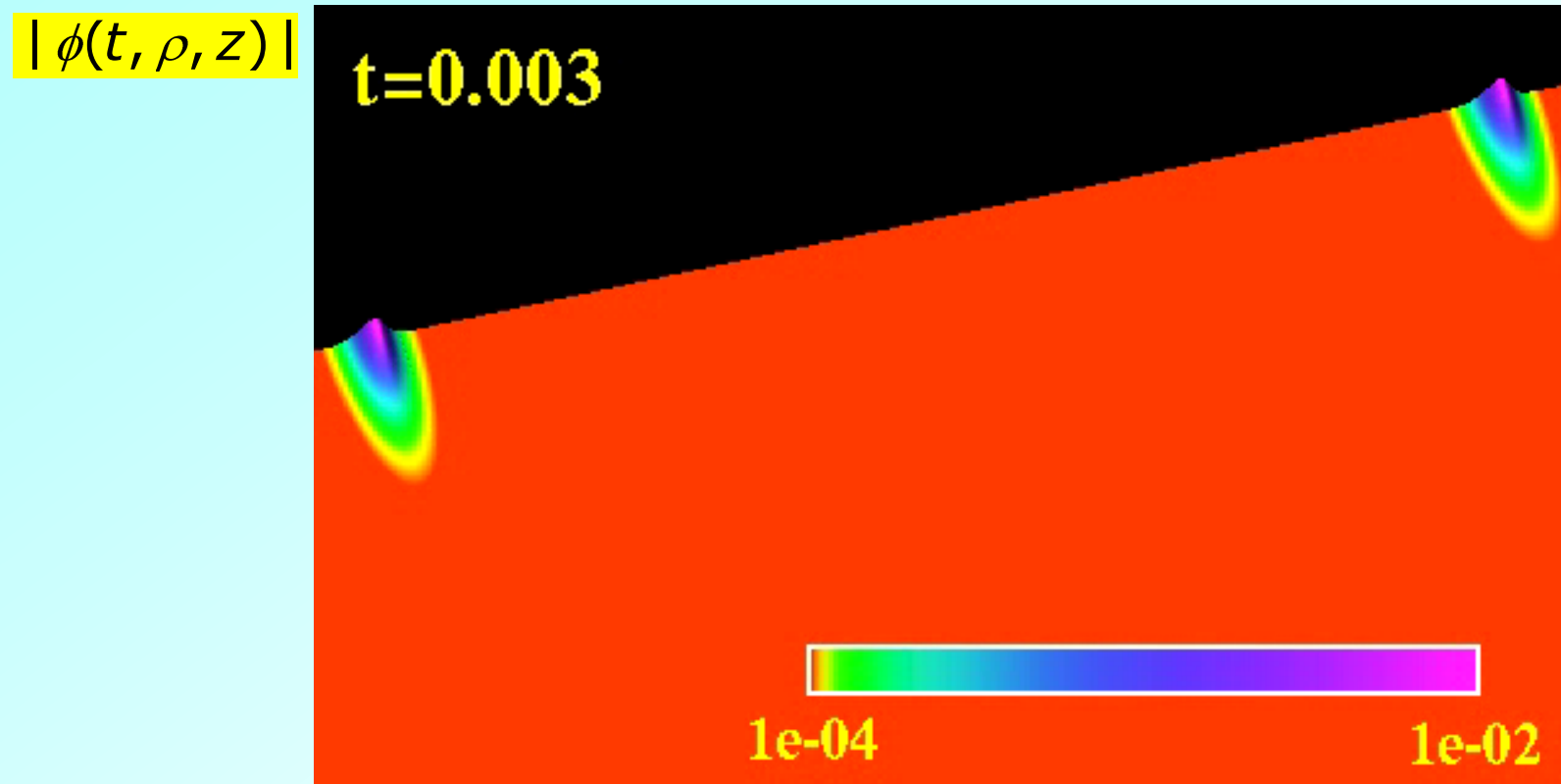
($\gamma = 4$)



Collision results in black hole formation

Head-on Collision of Relativistic Boson Stars II

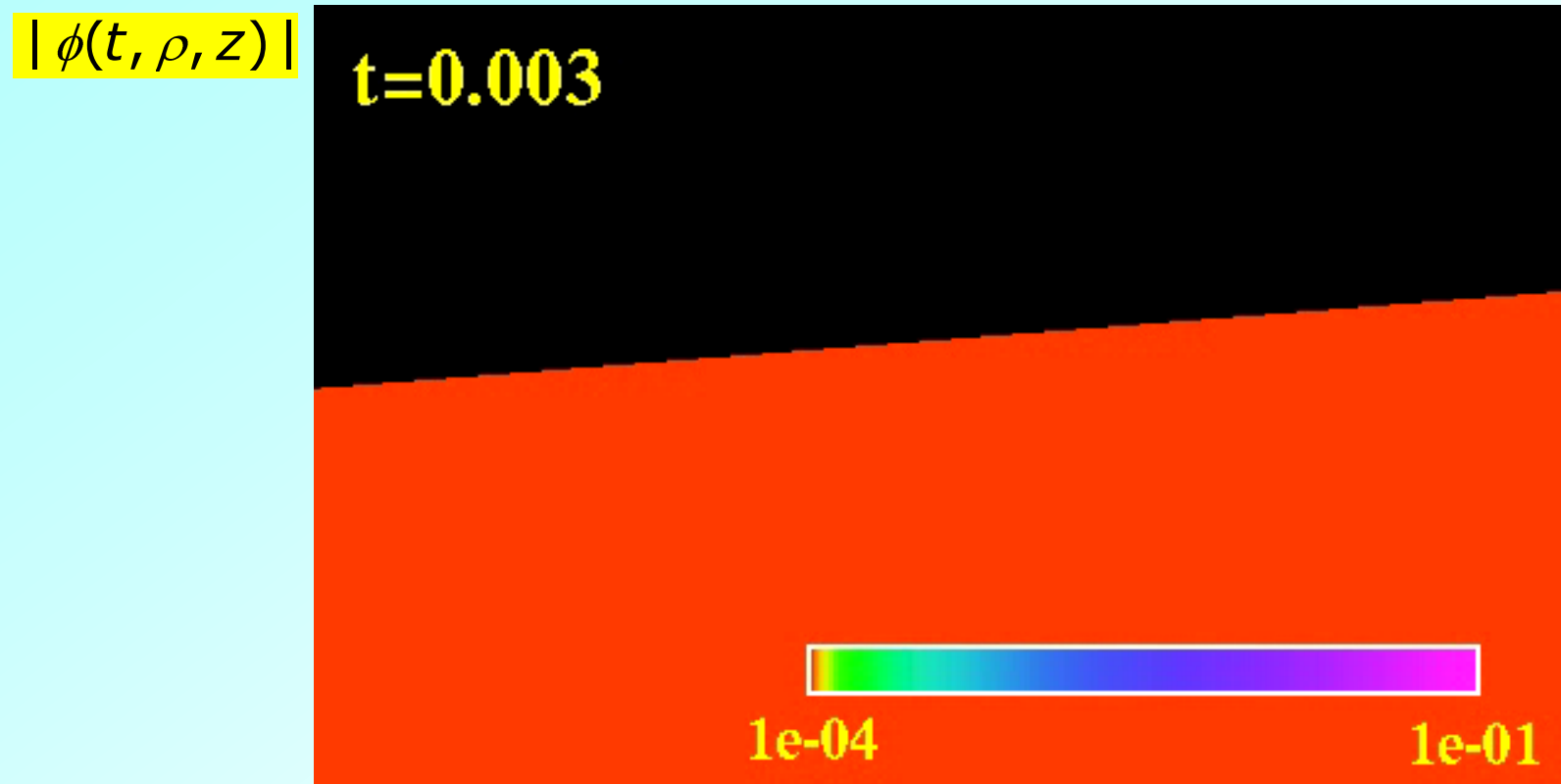
($\gamma = 3.125$)



Collision is relatively close to threshold, but sub-critical

Head-on Collision of Relativistic Boson Stars II

($\gamma = 3.125$ [detail])



So ...

- Do collisions of particles at sufficiently high energy necessarily form black holes?

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 - YES (which is what most people expected)

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So ...

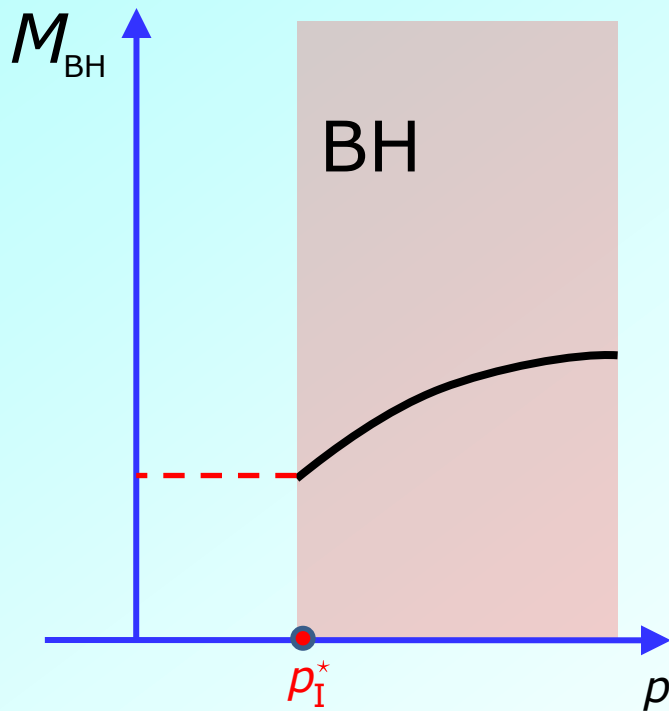
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- In fact, for case of mini-boson stars apparently form for γ about 1/3 what one might expect from naïve hoop-conjecture argument
- However, note:
 - Stable boson stars: $\max(2m(r) / r) \approx 0.5$
 - Massless scalar field (critical collapse): $\max(2m(r) / r) \approx 0.5$

Key Open Question ...

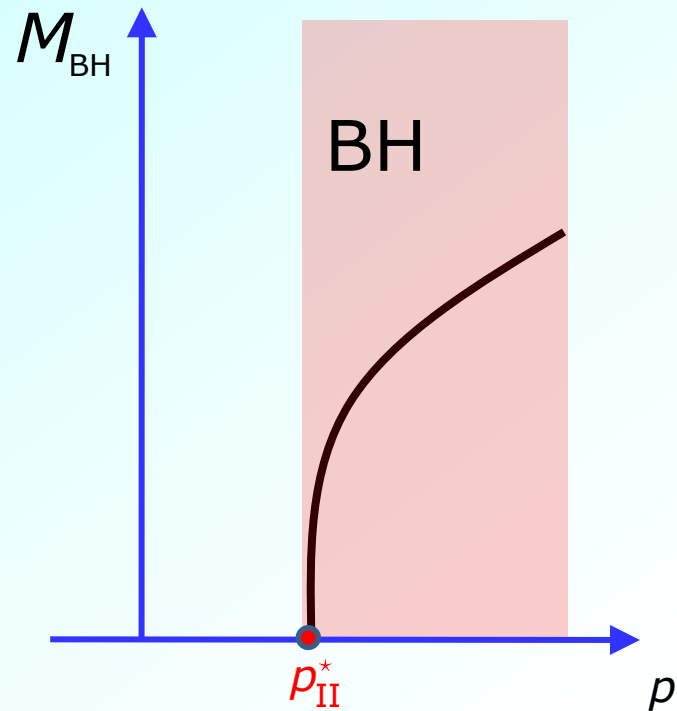
What happens at the *threshold* of black hole formation?

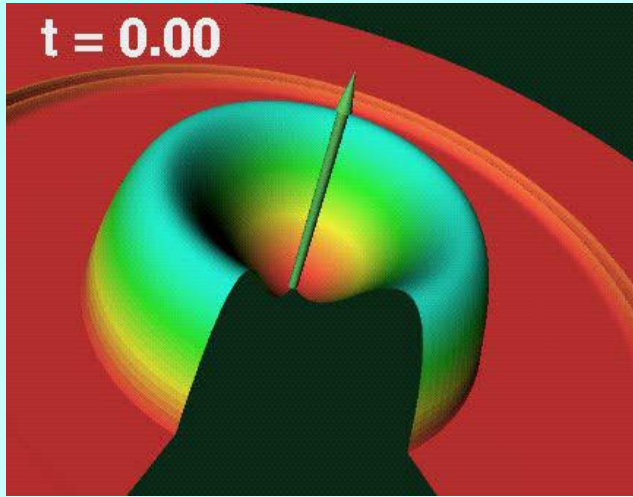
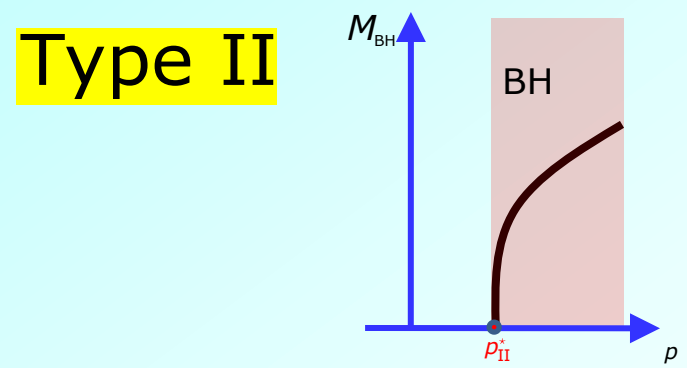
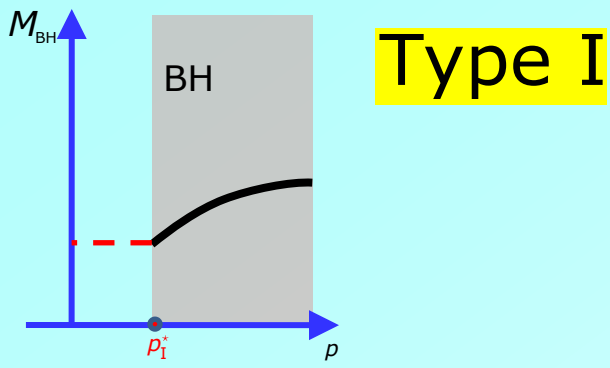
Types of Black Hole Transitions

Type I

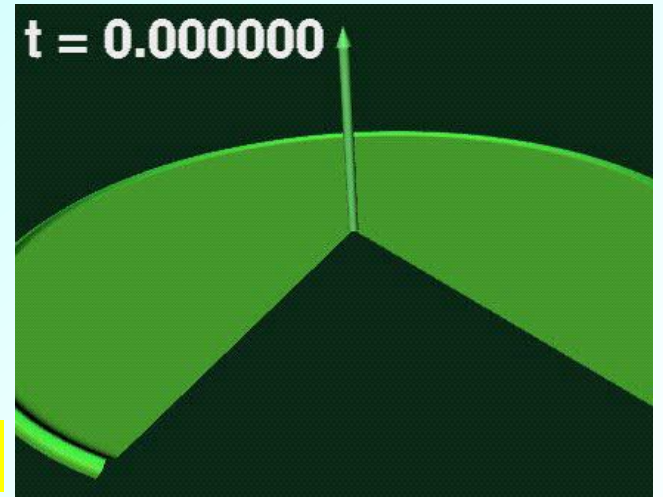


Type II

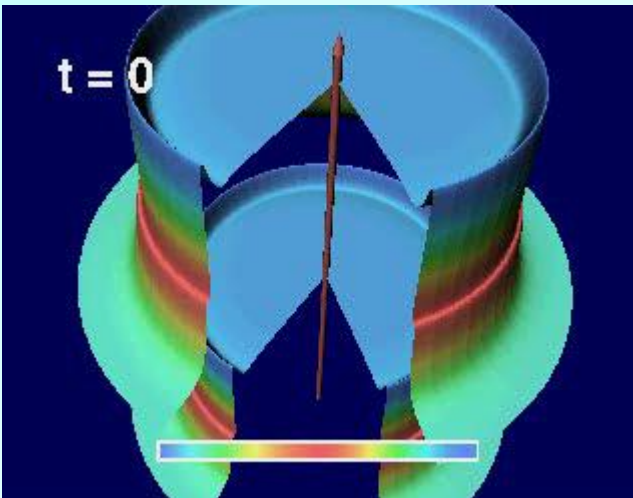




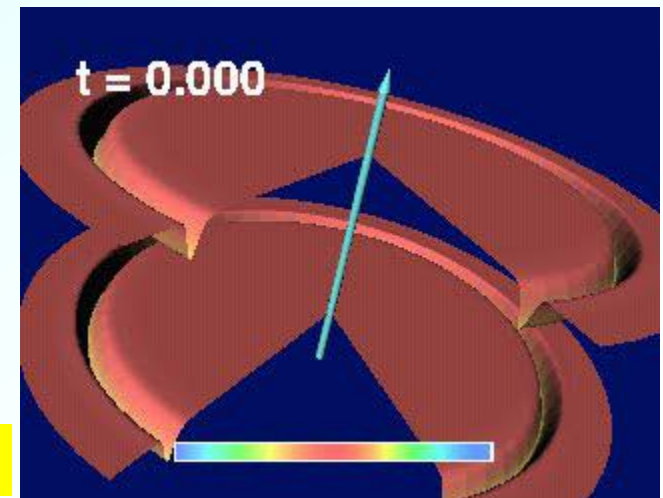
perfect fluid



massless scalar field

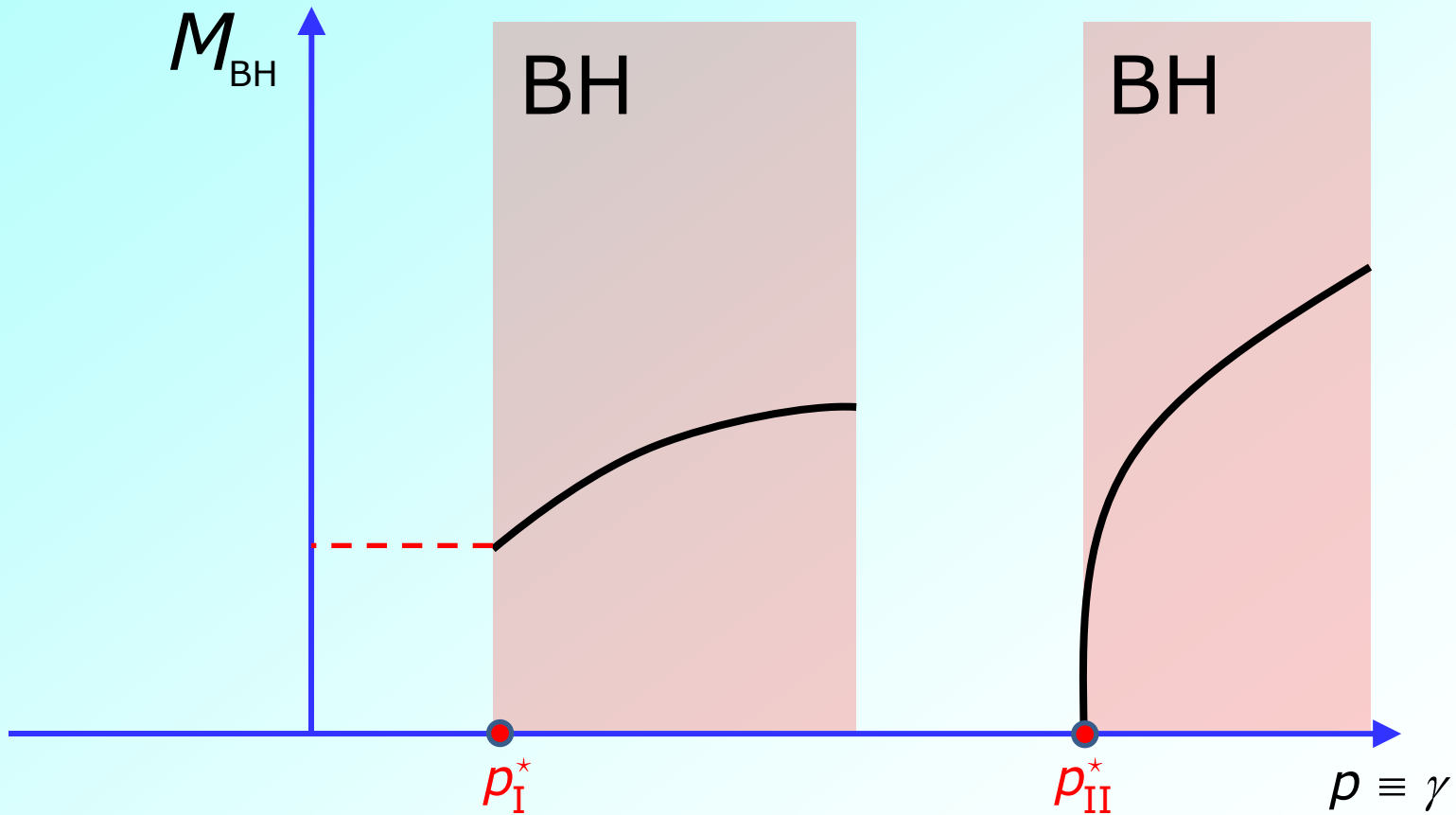


SU(2) Yang Mills field



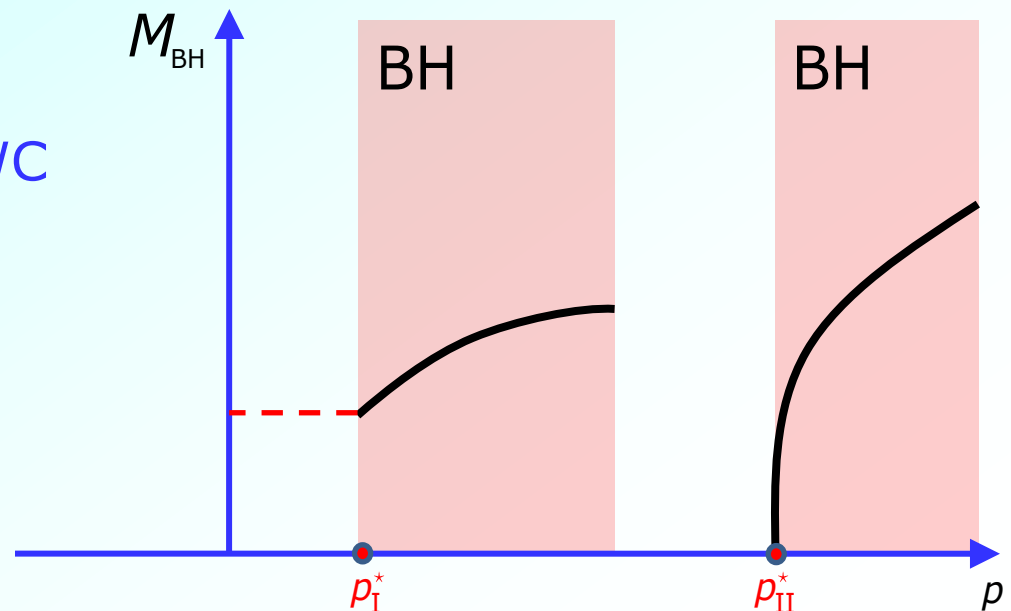
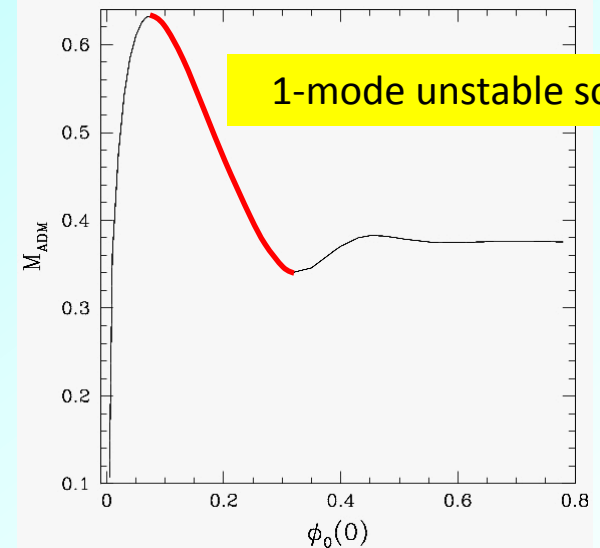
SU(2) Yang Mills field

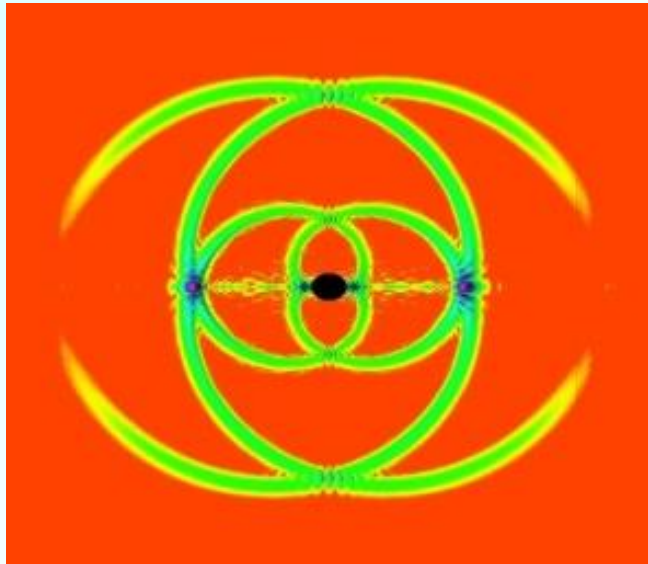
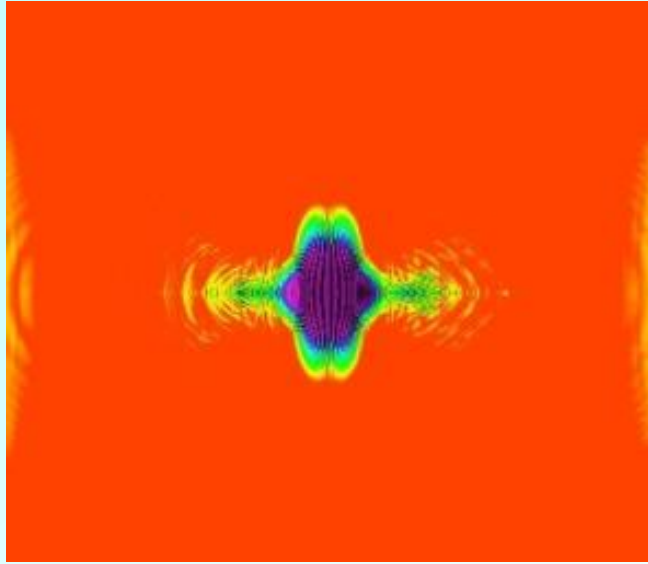
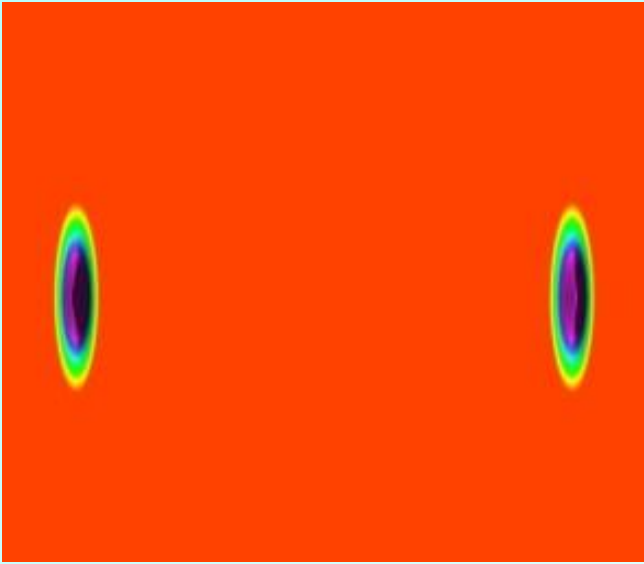
Conjectured "Phase Diagram" (Schematic)

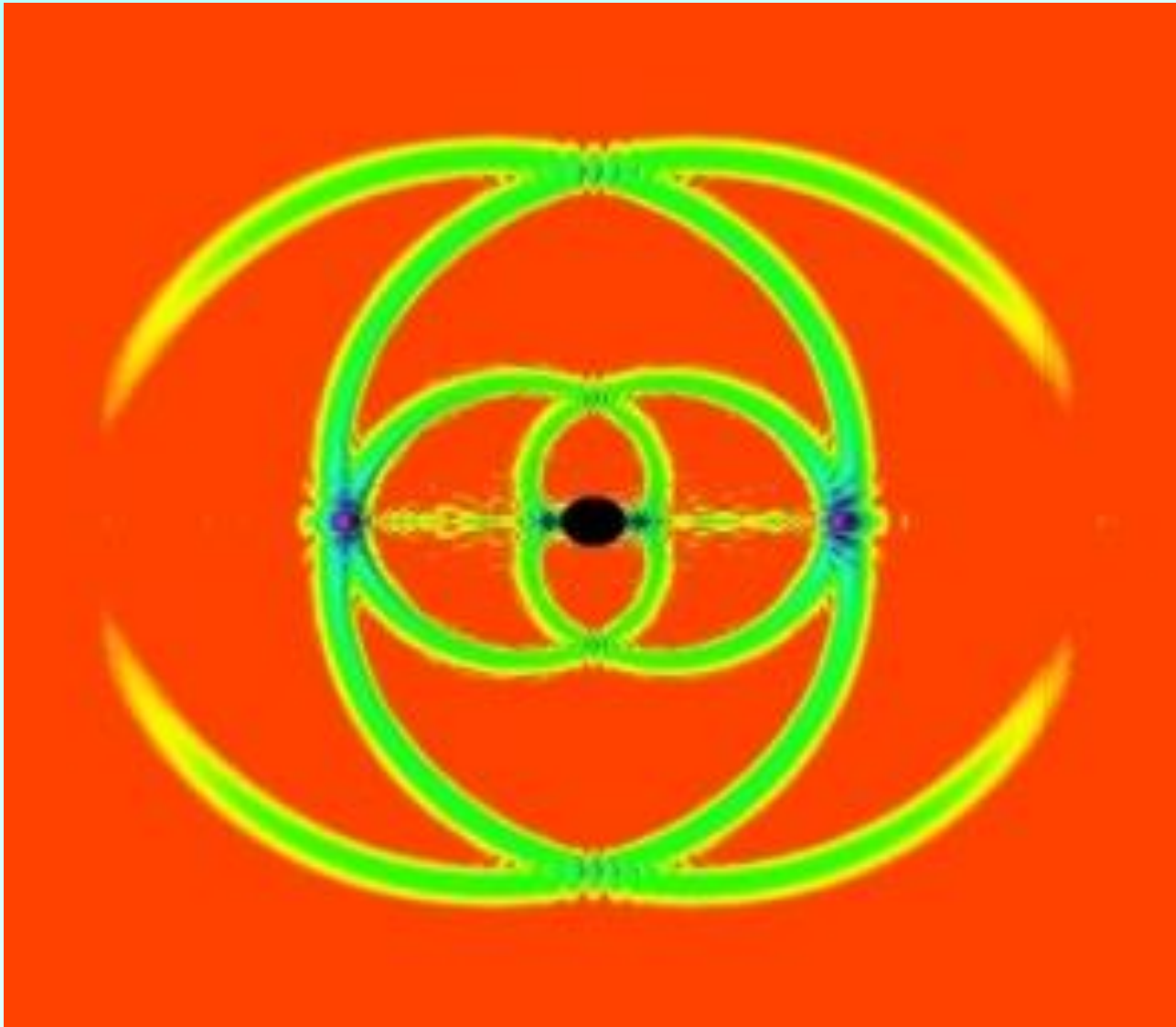


Threshold Solutions

- ρ_I^* Type I
 - “Perturbed”, unstable boson star
- ρ_{II}^* Type II
 - Spherically symmetric massless scalar ? (MWC 1993)
 - Axisymmetric vacuum Einstein (Abrahams & Evans 1993) ?
 - Something else ??







Planck scale may *not* be the end of short distance physics!

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- Use of Aichelburg-Sexl solution to approximate collision is well-justified, as is use of black holes themselves to investigate cross sections etc. for non-head-on particle collisions
- Expect results to be insensitive to details of matter model
- Questions remain concerning the threshold of black hole formation, with indications that strong gravity effects could generate structure below Planck scale (with tuning of initial conditions)

From blog of Science Online's recent (01-22)
article on calculations:

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article on calculations:

"Interesting how some articles just seem to draw
in crackpots from all over the net like a
supermassive black hole."