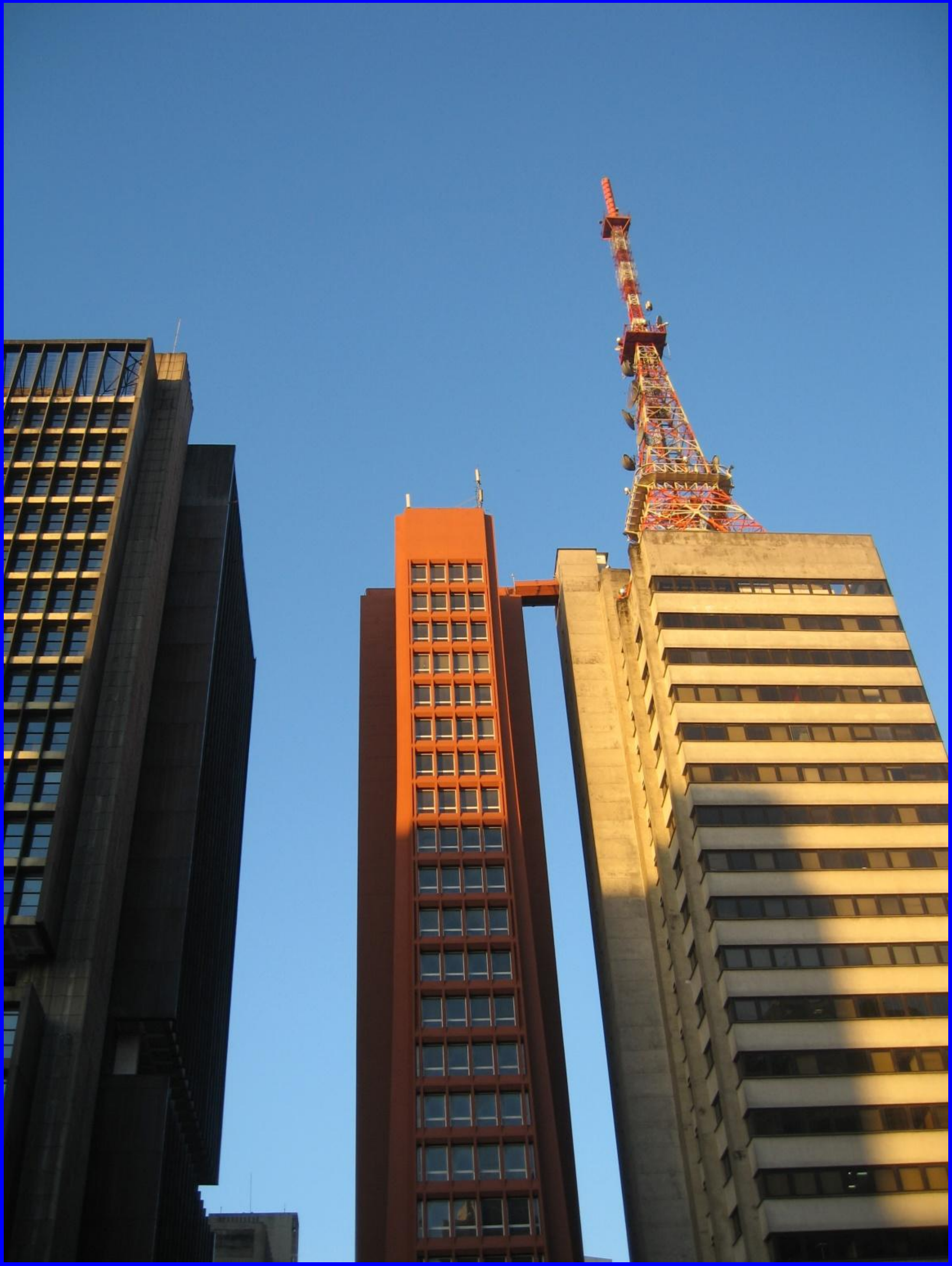


GENERAL RELATIVISTIC SIMULATIONS

Matthew W Choptuik

CIFAR Cosmology & Gravity Program

Dept. of Physics & Astronomy, UBC

Instituto de Fisica Teorica
Universidade Estadual Paulista

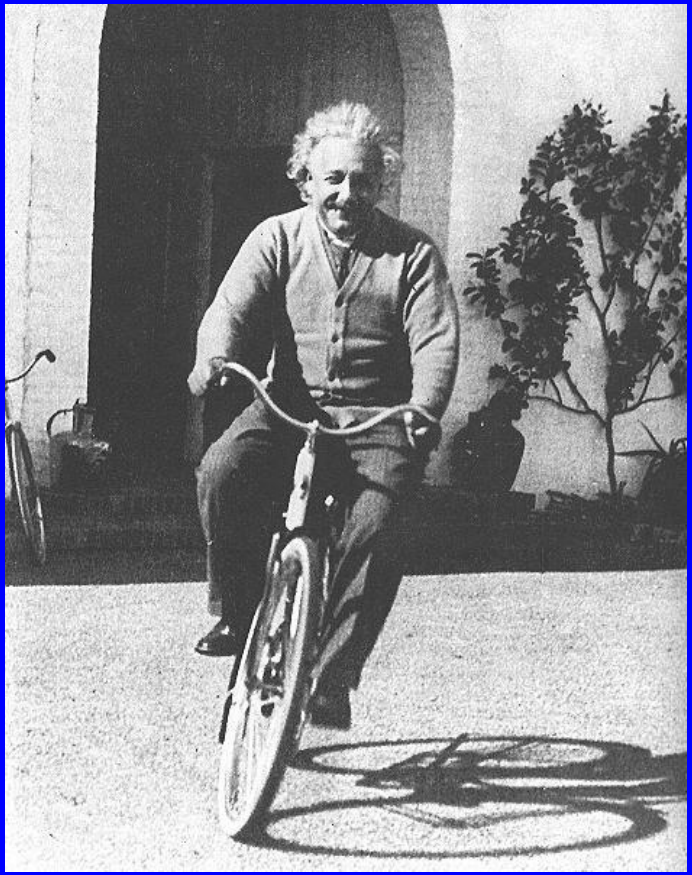
Sao Paulo, Brazil
March 8, 2010

Outline

- General relativity
  - Newtonian vs relativistic gravity
  - Strong field gravity
  - Numerical relativity: what and why
- General relativity as a Cauchy (initial value) problem
  - Traditional 3+1 approach
  - Well posedness
  - Well posed formulations
- Computational considerations
- Selected examples
  1. Interacting boson stars within conformally flat approximation (Mundim)
  2. Accretion of (magneto-)fluids onto a black hole (Penner)
  3. Black hole collisions (Pretorius)
- Future prospects

General Relativity

- Einstein (1916)
- Gravitational effects consequence of curvature of spacetime; curvature consequence of matter-energy distribution in spacetime
- Spectacular predictions
  - Expanding universe
  - Black holes
  - Worm holes
  - Gravitational waves



Newtonian Gravitation

- Gravitational force on object with gravitational mass m_g

$$\vec{F} = -m_g \vec{\nabla} \phi$$

$$\nabla^2 \phi \propto \rho$$

- Single Newtonian potential (single field) ϕ describes gravitational interaction
- Only objects with mass contribute to energy density ρ
- Action at a distance: Changes in gravitational field propagate instantaneously to rest of universe

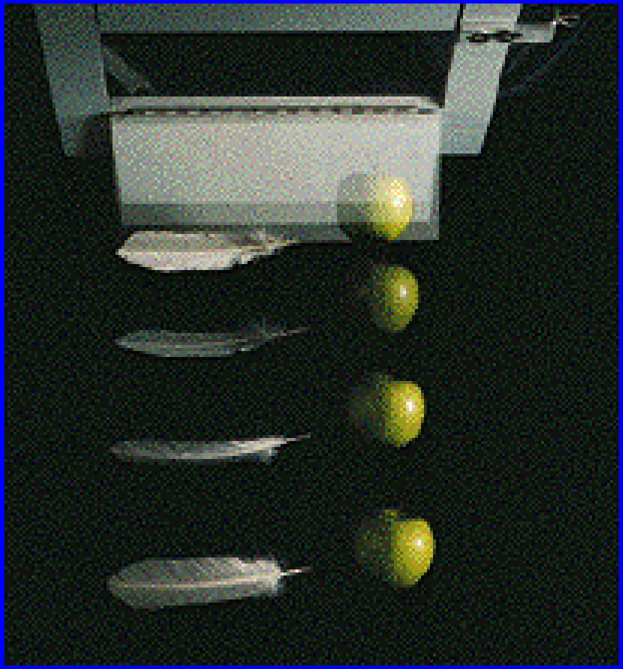
Newtonian Gravitation

Universality of Free Fall

Assuming that the inertial mass and the gravitational mass are proportional, with the same proportionality constant for all materials:

$$m_i \vec{a} = \vec{F} = -m_g \vec{\nabla} \phi$$

$$\vec{a} = -\nabla \phi$$



Relativistic Gravitation: General Relativity

Universality of free-fall elevated to Principle of Equivalence

- Locally, uniform gravitational field indistinguishable from uniform acceleration
- "Real" gravitational effects show up in non-uniformities of gravitational field (curvature of spacetime)
- Gravitational field much more complicated than in Newtonian case, essentially need four potentials plus two "wave fields"
- No action at a distance: disturbances in the gravitational field travel at most at the speed of light, c
- All forms of energy act as sources for gravitational field

The Metric

- The geometrical information about spacetime is completely encoded by the (symmetric) metric tensor

$$g_{\mu\nu}(x^\alpha) \equiv \begin{bmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ \bullet & g_{11} & g_{12} & g_{23} \\ \bullet & \bullet & g_{22} & g_{23} \\ \bullet & \bullet & \bullet & g_{33} \end{bmatrix}$$

- Spacetime distance (squared) between nearby events is given by

$$ds^2 = \sum_{\mu=0}^{3} \sum_{\nu=0}^{3} g_{\mu\nu} dx^\mu dx^\nu$$

General Relativity – Field Equations
($G = c = 1$)

- Einstein field equations

$$G_{\mu\upsilon} = R_{\mu\upsilon} - \tfrac{1}{2} R = 8\pi T_{\mu\upsilon}$$

$$R_{\mu\nu} = R_{\mu\upsilon}[\partial_\sigma \partial_\tau g_{\alpha\beta}, \partial_\sigma g_{\alpha\beta}, g_{\alpha\beta}] \qquad R = g^{\mu\upsilon} R_{\mu\upsilon}$$

$R_{\mu\upsilon}$ and R are linear in $\partial_\sigma \partial_\tau g_{\alpha\beta}$, but highly non-linear in $\partial_\sigma g_{\alpha\beta}, g_{\alpha\beta}$

- If matter fields are present, their equations of motion must be solved in concert with the Einstein equations

General Relativity—Strong Field Regime

- GR is an inherently non-linear theory: all forms of stress / energy / momentum, including those from the gravitational field itself contribute to spacetime curvature

- Highly non-trivial, dynamical solutions exist in the *vacuum* case

$$G_{\mu\upsilon} = 0$$

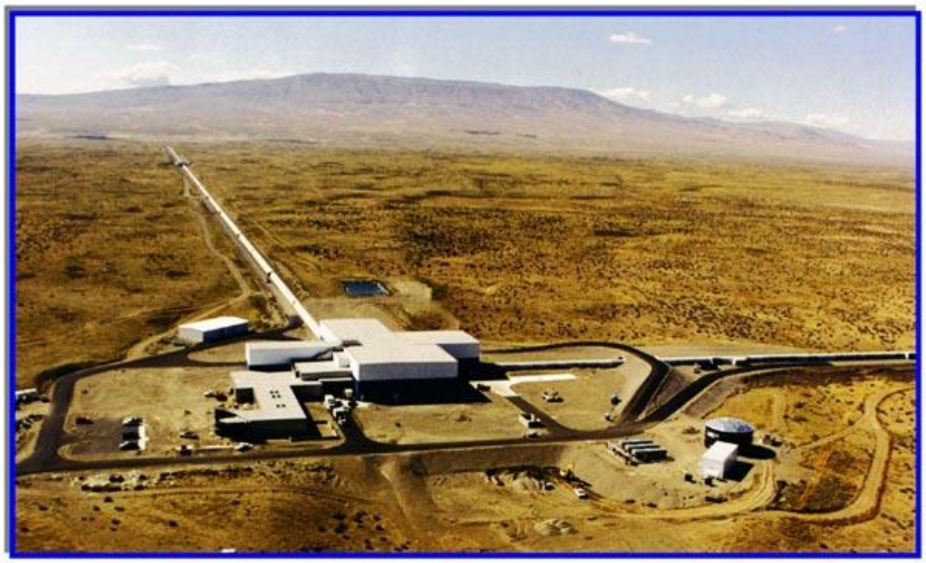
- Heuristically, two dimensionless parameters characterize strong-field regime

$$\frac{M}{L} = \frac{G}{c^2}\frac{M}{L} = \frac{\text{mass of system}}{\text{characteristic length scale of system}}$$

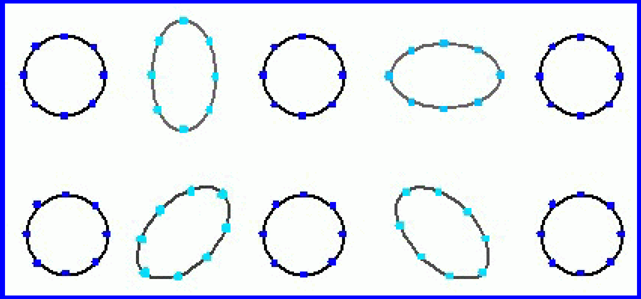
$$\frac{v}{c} = \frac{\text{characteristic internal velocities}}{\text{speed of light}}$$

Gravitational Radiation

- Gravitational waves: "ripples" in the curvature of spacetime
- At least in weak field limit, very much analogous to electromagnetic radiation; propagate at speed of light, transverse, two polarizations, frequency set by dynamics of source



The Laser Interferometer Gravitational Wave Observatory (LIGO) installation near Hanford WA. Each interferometer arm is 4 km long. A similar instrument is located near Livingston LA (www.ligo.caltech.edu)



Cause periodic, quadrupolar variations in distance between freely falling objects (or induce strains in objects with interactions)

Sources of Gravitational Radiation

- For efficient radiation, need (large) masses confined to regions comparable in size to their Schwarzschild radii, R_S

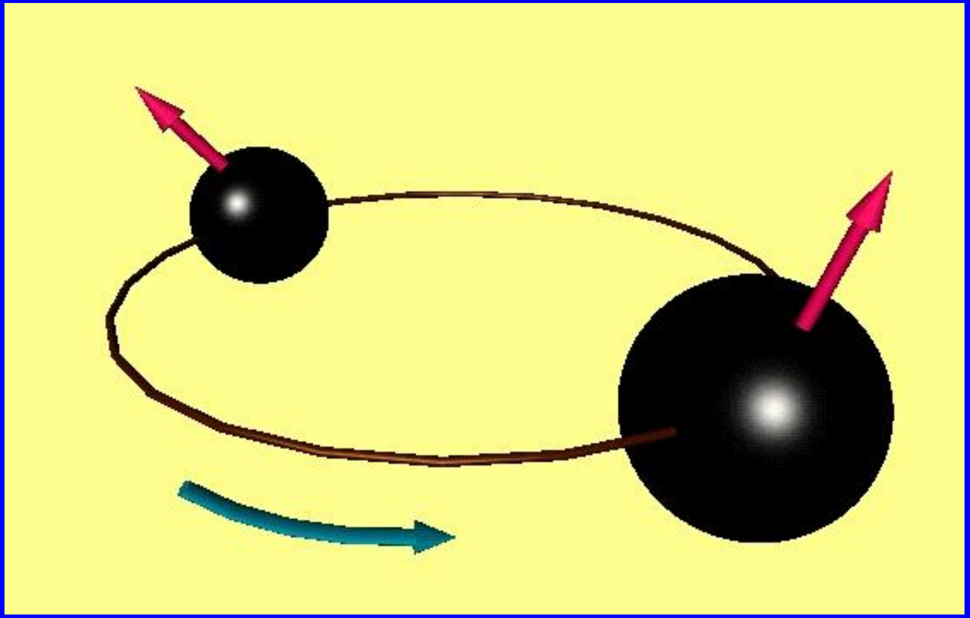
$$R_S = \frac{2G}{c^2} M$$

$$\frac{2G}{c^2} = 1.5 \times 10^{-27} \frac{\text{m}}{\text{kg}} = 3.0 \frac{\text{km}}{M_{\text{Sun}}}$$

- R_S for Earth is about 1 cm!

- Also need redistribution of significant fraction of mass-energy at close to light speed

- Compact binary systems (BHs, neutron stars good candidates)



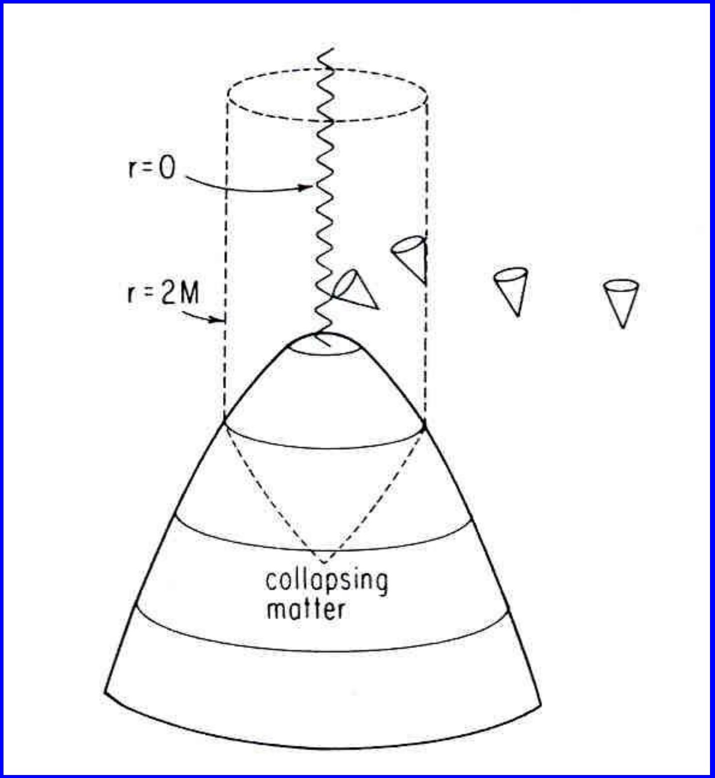
LIGO design sensitivities: (30-1000 Hz)

Phase 1: $\delta L / L \approx 10^{-21}$

Phases 2-3: $\delta L / L \approx 10^{-23}$

Gravitational Collapse and Black Holes

- Black hole: Region of spacetime from which no physical signal can escape
- During collapse of matter and/or radiation, BH forms when gravitational field becomes strong enough to "trap" light rays
- Surface of black hole is known as the event horizon
- Singularities (infinite, crushing gravitational forces) inevitable inside black holes



(From Wald, *General Relativity*, 1984)

Cosmic censorship hypothesis (Penrose): Singularities resulting from gravitational collapse of "reasonable" matter are generically hidden inside black holes; collapse does not generically produce "naked singularities"

Why Numerical Relativity?

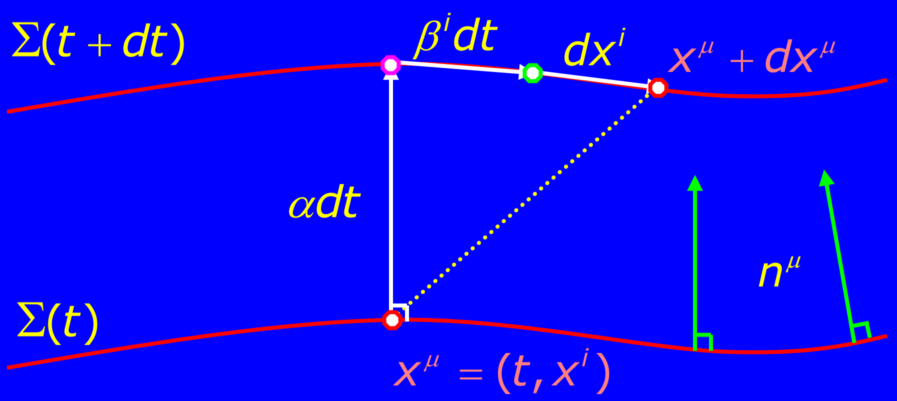
- DEFINITION: Computational solution of Einstein field equations for metric tensor, plus (field) equations of motion for any matter fields that have been coupled to gravity
- Motivation from several different areas
  - Astrophysics
  - Fundamental gravitational physics
  - Applied mathematics
  - Computational science
- Difficult to make progress solving Einstein equations using traditional "closed form" ("analytic") techniques – in principle numerical relativity allows most general / realistic cases to be studied

Numerical Relativity: Key Challenges

- Formulation and discretization of equations of motion
- Singularity avoidance when evolving BH spacetimes
- Computational demands
- NUMERICAL STABILITY
- Tie-in to observations (gravitational wave extraction)
- Shortage of personnel (lots of opportunities for new efforts!)

General Relativity: The Cauchy Problem
(3+1 / ADM formalism)

- View spacetime as stack of 3-dim. spacelike hypersurfaces, labeled by time parameter, t



$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$
$$= -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dx^i)(dx^j + \beta^j dx^j)$$

- Kinematical variables
  - lapse fcn, α
  - shift vector, β^i
  - Represent coordinate (gauge) freedom of theory
  - Must be specified/fixed (explicitly or implicitly)

- Dynamical variables
  - 3-metric, γ_{ij}
  - extrinsic curvature, $K_{ij} = -\nabla_i n_j \sim \partial_t \gamma_{ij}$
  - Describe intrinsic geometry of hypersurfaces, as well as how surfaces are embedded in spacetime

SPACETIME = TIME HISTORY OF THE GEOMETRY OF AN INITIAL SPACELIKE HYPERSURFACE (GEOMETRODYNAMICS; Wheeler)

General Relativity: The Cauchy Problem

- Einstein equations decompose into two classes

1. Evolution equations; schematically

$$E_{ij}[\partial_t^2 g_{ij},\ \partial_k \partial_l g_{ij},\ \partial_k K_{ij},\ K_{ij},\ g_{ij}] = e_{ij}$$

   Typically "hyperbolic" in a given coordinate system

2. Constraint equations, schematically

$$H_\mu[\partial_k \partial_l g_{ij},\ \partial_k K_{ij},\ K_{ij},\ g_{ij}] = h_\mu$$

   Typically "elliptic" in a given coordinate system

- Constraint equations must be satisfied on each slice, including the initial slice; evolution equations preserve constraints in time (direct analogy with Maxwell equations)

Einstein Equations: Traditional 3+1 Form

1. Constraint equations

$$R + K^2 - K_{ij}K^{ij} = 16\pi\rho$$

$$D_i K^{ij} - D^j K = 8\pi j^i$$

2. Evolution equations

$$L_t \gamma_{ij} = L_\beta \gamma_{ij} - 2\alpha K_{ij}$$

$$L_t K_{ij} = L_\beta K_{ij} - D_i D_j \alpha + \alpha\left(R_{ij} + KK_{ij} - 2K_{ik}K^k{}_j\right) - 8\pi\alpha\left(S_{ij} - \frac{1}{2}\gamma_{ij}\left(S - \rho\right)\right)$$

$R_{ij} \equiv$ 3-Ricci tensor

$R \equiv \gamma^{ij} R_{ij}$

$\mathrm{K} \equiv \gamma^{ij} K_{ij}$

$D_i \equiv$ 3-covariant derivative

$L_n \equiv$ Lie derivative w.r.t. n

$\rho \equiv$ energy density

$j^i \equiv$ momentum density

$S_{ij} \equiv$ 3-stress tensor

$S \equiv \gamma^{ij} S_{ij}$

Well-Posedness

- Need Cauchy problem to be well posed

- Roughly: given initial data $\gamma_{ij}(0, x^i)$ and $K_{ij}(0, x^i)$ satisfying constraints, do $\gamma_{ij}(t, x^i)$ and $K_{ij}(t, x^i)$ remain bounded in time (provided no physical singularities develop)?

- Essentially a statement of the stability of the solutions of the equations of motion and can be completely unrelated to stability of underlying physics; i.e. formulation of equations can be pathological in same sense as attempting to solve a heat/diffusion with negative diffusion coefficient

- Standard 3+1 form of Einstein equations *not* well posed in general!

- (Numerical) solutions can be expected to "blow up" even for initial data that should lead to globally regular/bounded continuum solutions

- Many "pioneering" 3D (D refers to number of spatial dimensions on which fields/functions have non-trivial dependence) numerical relativity efforts (early '90's) were doomed due to this fact

Well-Posed Formulation I: BSSN
(Baumgarte/Shapiro/Shibata/Nakamura)

- Analysis of characteristic structure (hyperbolicity) of standard 3+1 form suggests that mixed spatial derivatives of the 3-metric in the R_{ij} term are responsible for difficulty

- BSSN approach circumvents this problem through introduction of auxiliary variables which are functions of the first spatial derivatives of the 3-metric

- Also employs "conformal" decomposition techniques pioneered by Lichnerowicz in the 1940's and developed in the 70's and 80's by York and others

  - Can view as a "spin decomposition" of dynamical degrees of freedom of the gravitational field

    - Spin-0 (trace), spin-1 (longitudinal) pieces: related to coordinate invariance (gauge symmetry of theory), conservation laws (conservation of energy, 3-momentum)
    - Spin-2 (transverse traceless): related to radiative degrees of freedom

  - Transforms messy general differential operators into simpler, well-behaved ones

BSSN Approach

- Definitions

unimodular conformal 3-metric: $\tilde{\gamma}_{ij} \equiv e^{-4\phi}\gamma_{ij}$ $(\det \tilde{\gamma}_{ij} = 1)$

conformal function: ϕ

trace-free part of extrinsic curvature: $A_{ij} \equiv K_{ij} - \frac{1}{3}\gamma_{ij}K$

conformally scaled extrinsic curvature: $\tilde{A}_{ij} \equiv e^{-4\phi}A_{ij}$

conformal connection functions: $\tilde{\Gamma}^i \equiv \tilde{\gamma}^{jk}\tilde{\Gamma}^i{}_{jk} \equiv -\partial_j\tilde{\gamma}^{ij}$

- Equations of motion take the form

$$
\begin{aligned}
\partial_t\phi &= S_\phi \\
\partial_t K &= S_K \\
\partial_t\tilde{\gamma}_{ij} &= S_{\tilde{\gamma}} \\
\partial_t\tilde{A}_{ij} &= S_{\tilde{A}} \\
\partial_t\tilde{\Gamma}_i &= S_{\tilde{\Gamma}}
\end{aligned}
$$

where the source functions, *S*, do not explicitly involve the mixed second spatial derivative terms

Well Posed Formulation II: Generalized Harmonic
(Friedrich, Garfinkle, Pretorius)

- Harmonic coordinates

$$\nabla^{\alpha}\nabla_{\alpha}x^{\mu} = 0$$

- Yields well-posed formulation of Einstein equations, used to great advantage by Choquet-Bruhat in ’50’s in proving local existence and uniqueness of solutions
- Sacrifices too much coordinate freedom: once coordinates and time derivatives are fixed at *t=0*, they are fixed for all future and past times
- Minimal flexibility to adapt coordinates, particularly time coordinate (slicing condition), in response to evolution
- Coordinate singularities tend to develop, especially when gravitational field is strong (black holes, e.g.)

Generalized Harmonic (cont.)

- IDEA: Choose

$$\nabla^{\alpha}\nabla_{\alpha}x^{\mu} = H^{\mu}$$

where gauge source functions, H^{μ}, are viewed as specified quantities, with no explicit dependence on second derivatives of the metric

- Einstein equations become

$$g^{\gamma\delta}g_{\alpha\beta,\gamma\delta} + 2g^{\gamma\delta}{}_{,(\alpha}g_{\beta)\delta,\gamma} + 2H_{(\alpha,\beta)} - 2H_{\delta}\Gamma^{\delta}_{\alpha\beta} + 2\Gamma^{\gamma}_{\delta\beta}\Gamma^{\delta}_{\gamma\alpha} + 8\pi\left(2T_{\alpha\beta} - g_{\alpha\beta}T\right) = 0$$

- System prone to instabilities ("constraint violating modes"), add constraint damping terms

$$g^{\alpha\beta}g_{\mu\nu,\alpha\beta} + \ldots + \kappa\left(n_{\mu}C_{\nu} + n_{\nu}C_{\mu} - g_{\mu\nu}n^{\alpha}C_{\alpha}\right) = 0$$

$C^{\mu} \equiv H^{\mu} - \nabla^{\alpha}\nabla_{\alpha}x^{\mu}$

$n_{\mu} \equiv$ unit normal to $t = $ const. slices

$\kappa \equiv$ adjustable parameter

Computational Considerations

- Smoothness of solutions plays a key role
  - Determines how effective numerical solutions are, and how efficiently they can be computed
  - Dictates appropriate numerical approaches
- Types of solutions
  - Everywhere smooth (scalar, vector, tensor fields)
    - Finite difference, finite element, finite volume, spectral
    - In principle can achieve exponential convergence as function of computational work, *W*
  - Piecewise smooth (compressible fluids – shocks , but flow non-turbulent)
    - Finite volume methods based on integral formulation of equations and weak solutions preferred
    - Careful attention to characteristic structure required
    - Convergence typically power law in *W*, with reduced rate near shocks

Computational Considerations

- Types of solutions (cont.)
  - Non-smooth (turbulent flows)
    - Extremely challenging to simulate, many open questions, no completely satisfactory approach
- Adaptive mesh refinement (AMR)
  - Significant range in relevant spatial/temporal scales (factor of 100 or more for BH collisions)
  - Need methods where the discretization scale (mesh space) is allowed to vary from place to place in the solution domain in response to development of solution features
  - Is now routinely used in multi-dimensional numerical relativity computations and has been instrumental in allowing most of the 3D calculations to be performed at all
  - Will show example of the technique later

Computational Considerations

- Parallelization and high performance computation
  - Finite difference and finite volume codes readily parallelized (in principle), due to locality of interactions
  - Exhibit good scaling: time to run fixed computation on N processors (cores) goes like $1/N$
  - Typical calculations now run on 100’s to 1000’s of processors for several days
  - Software packages available to shield “users” from details of parallel implementation (CACTUS, PARAMESH, PAMR/AMRD, …)
- Symbolic computation
  - Equations of motion for mutli-dimensional computations in numerical relativity tend to be extremely complicated
  - Deriving, discretizing, implementing are all error-prone processes
  - Symbolic computation packages / techniques can be used to great advantage

Selected Examples

Bruno Mundim

BSc, MSc: University of Brasilia

PhD Thesis: UBC 2010

Numerical Studies of Boson Star Binaries

Current Position

PDF at Rochester Institute of Technology with Manuela Campanelli

Boson Stars

- Coupled Einstein-Klein-Gordon equations

$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$

$$T_{\mu\nu} = \frac{1}{2}(\phi_{,\mu}\phi^{\star}_{,\nu} + \phi^{\star}_{,\mu}\phi_{,\nu}) - \frac{1}{2} g_{\mu\nu}\phi_{,\alpha}\phi^{\star,\alpha} - g_{\mu\nu}V$$

$$V = \frac{1}{2}m^2 |\phi|^2$$

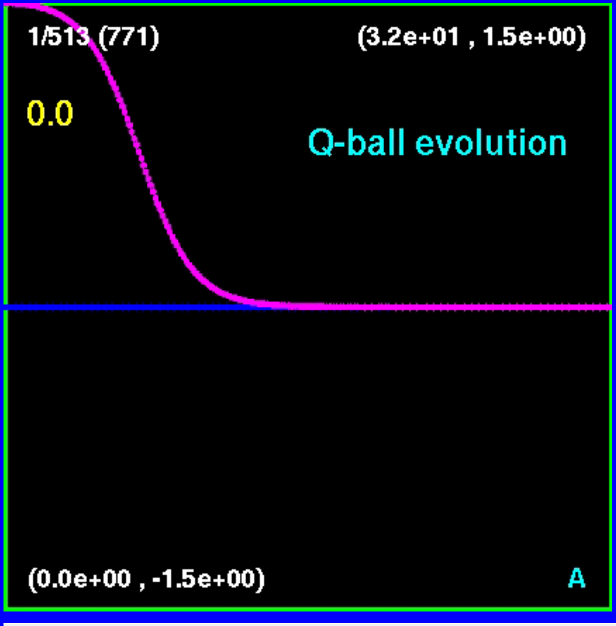
$$\nabla^{\alpha}\nabla_{\alpha}\phi = \frac{dV}{d\phi} = m^2\phi$$

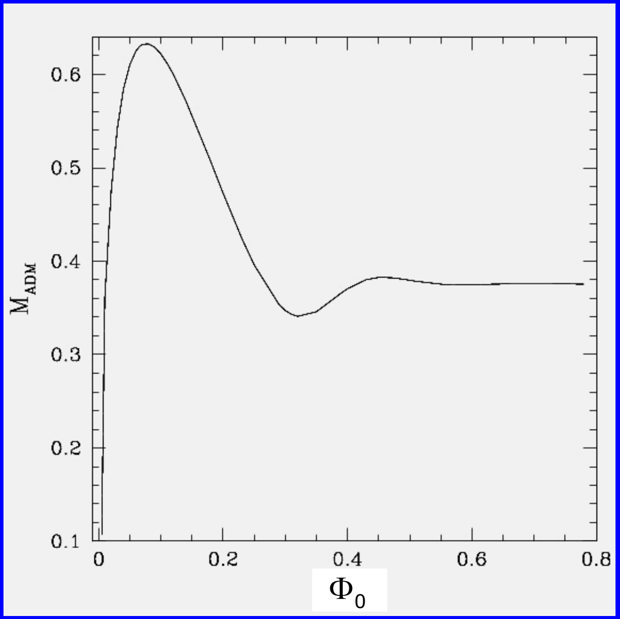
- Find spherically symmetric, stable, localized solutions by choosing "Schwarzschild-like" coordinates

$$ds^2 = -\alpha(r)^2 dt^2 + a(r)^2 dr^2 + r^2 d\Omega^2$$

and adopting ansatz

$$\phi(t,r) = \Phi(r) e^{i\omega t}$$





Conformally Flat Approximation to Einstein Equations

- 4-metric can be written as

$$g_{\mu\nu} = \begin{bmatrix} -\alpha^2 + \beta_k \beta^k & \beta_j \\ \beta_i & \gamma_{ij} \end{bmatrix}$$

- Conformally flat approximation: Demand

$$\gamma_{ij} = \psi^4 f_{ij}$$

where f_{ij} is the fixed flat metric and ψ is the conformal factor

- Time slicing condition

$$K \equiv \gamma^{ij} K_{ij} = 0$$

Conformally Flat Approximation (cont.)

- Unknowns / Equations

ψ : Hamiltonian constraint

$$\nabla^2 \psi = S_\psi$$

β_i : Momentum constraint

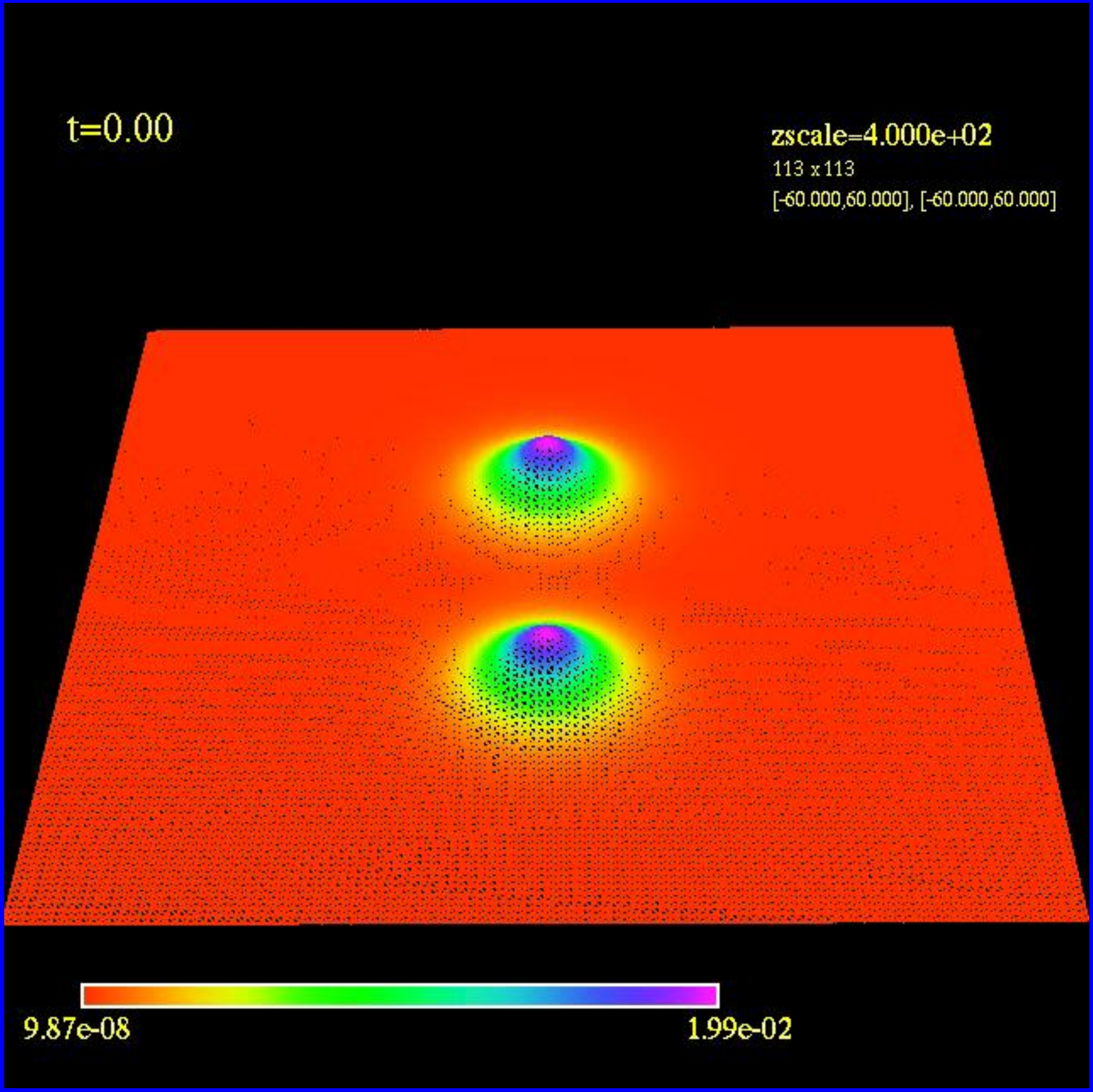
$$\nabla^2 \beta_i = S_{\beta_i}$$

α : Slicing condition

$$\nabla^2 \alpha = S_\alpha$$

- Solve together with Klein-Gordon equation for scalar field – no independent dynamics of gravitational field (no use of evolution equations for extrinsic curvature, K_{ij})

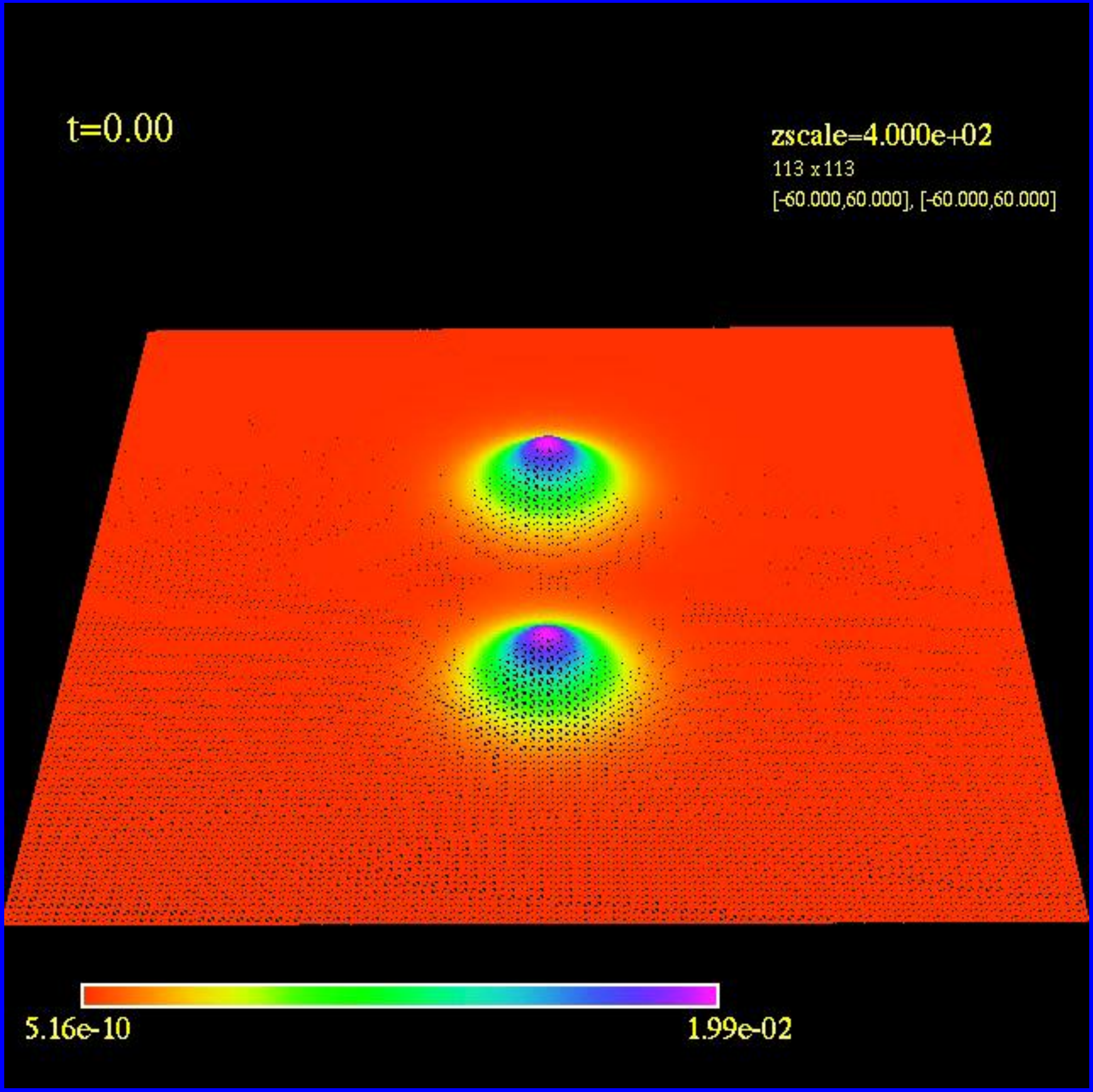
Interacting Boson Stars
1. Merger Forming Rotating Star



$v_0 = 0.7$

Interacting Boson Stars
2. Long Term Orbital Motion

$v_0 = 0.9$



Jason Penner

PhD Thesis: UBC 2010

Numerical Analysis of General Relativistic Magnetohydrodynamics

Current Position

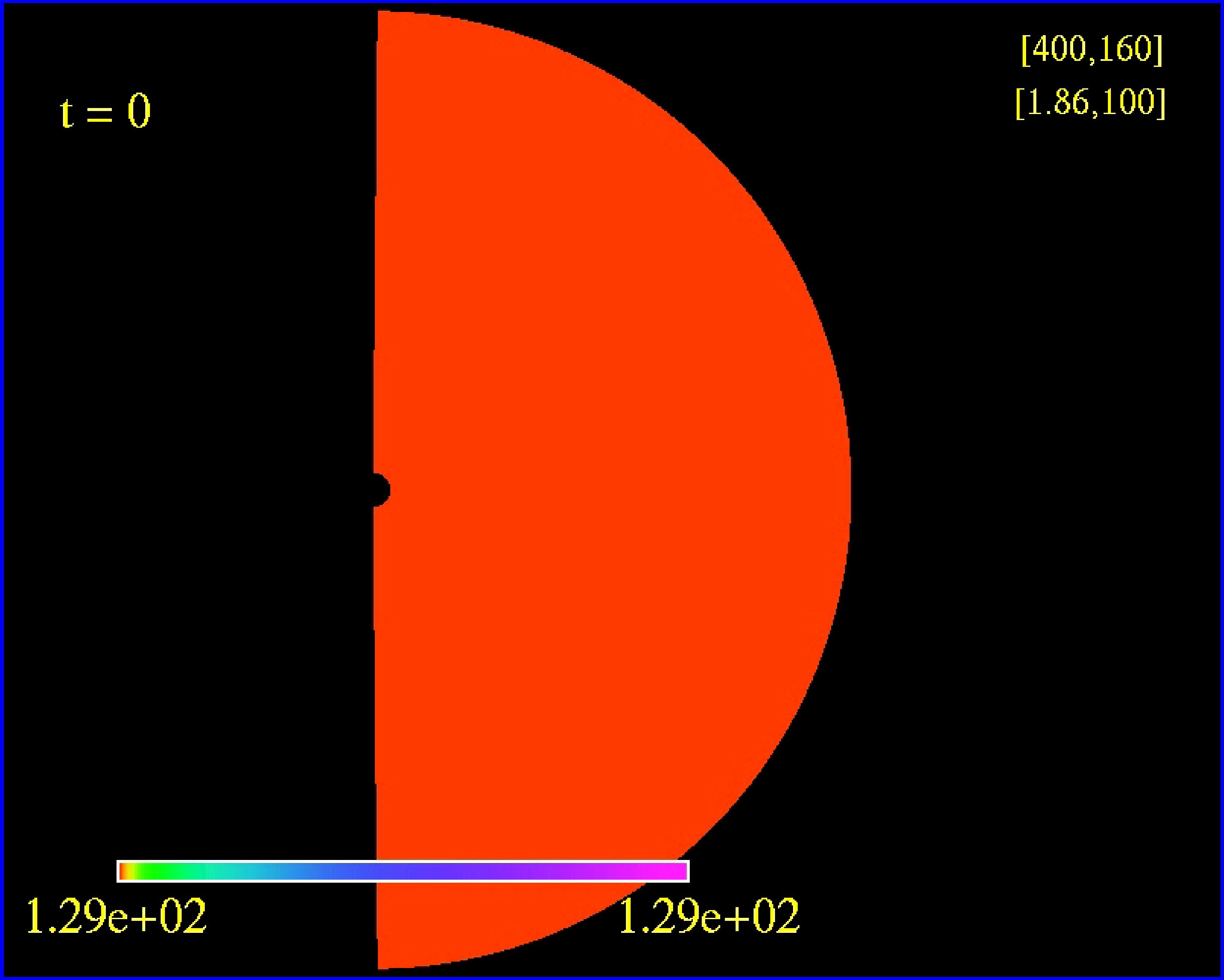
PDF at Southampton University with Nils Andersson

(Magneto)-Hydrodynamic Accretion onto Black Holes

- Solves equations of general-relativistic (magneto)-hydrodynamics for scenarios describing wind accretion onto a black hole
  - Geometry is fixed (Schwarzschild/Kerr for non-rotating/rotating)
  - Black hole moves at constant velocity through uniform fluid
- To reduce computational cost considers following “2D” problems
  - Axisymmetric accretion
  - Thin disk accretion
- Uses finite-volume, Gudonov-type methods (High Resolution Shock Capturing (HRSC) schemes), so that shocks and other types of waves (rarefaction, contact discontinuities) are accurately treated

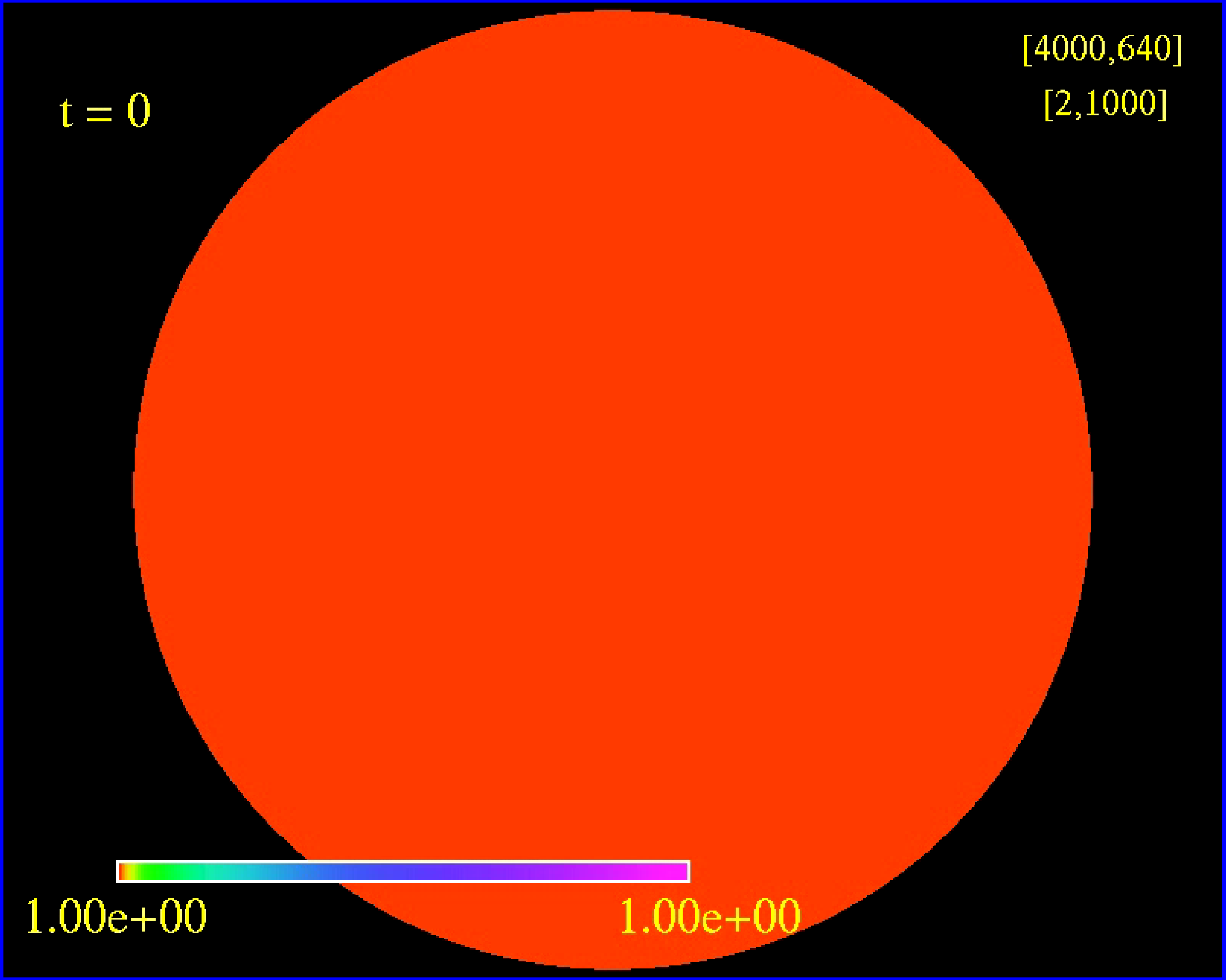
Axisymmetric Accretion (Unmagnetized)

$\rho(t,r,\theta)$



Thin Disk Accretion (Magnetized)

$P(t, r, \theta)$



Frans Pretorius

PhD Thesis: UBC 2002

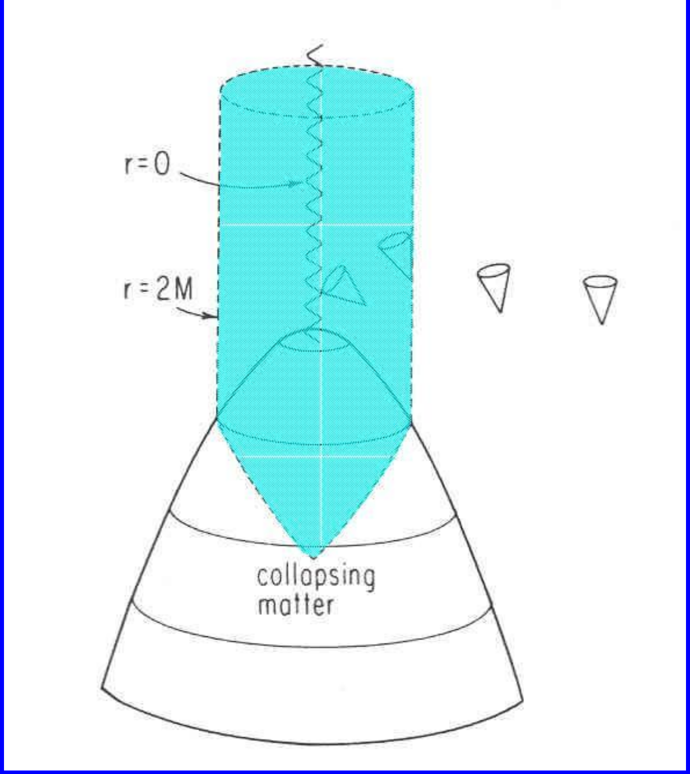
Numerical Simulations of Gravitational Collapse

Current Position

Assistant Prof. Dept of Physics, Princeton Univ.

Singularity Avoidance: Black Hole Excision

- To avoid singularity within black hole, exclude interior of hole from computational domain (Unruh)
- Problem: event horizon is globally defined, location unknown until complete spacetime geometry is in hand
- Apparent horizon functions as "instantaneous horizon" can be located at any instant of time
- Excise somewhat within apparent horizon
- Used in calculations showed here
- NOTE: "Moving punctures" technique is an alternate approach that has also been highly successful



Pretorius’ Black Hole Collision Code
Key Features

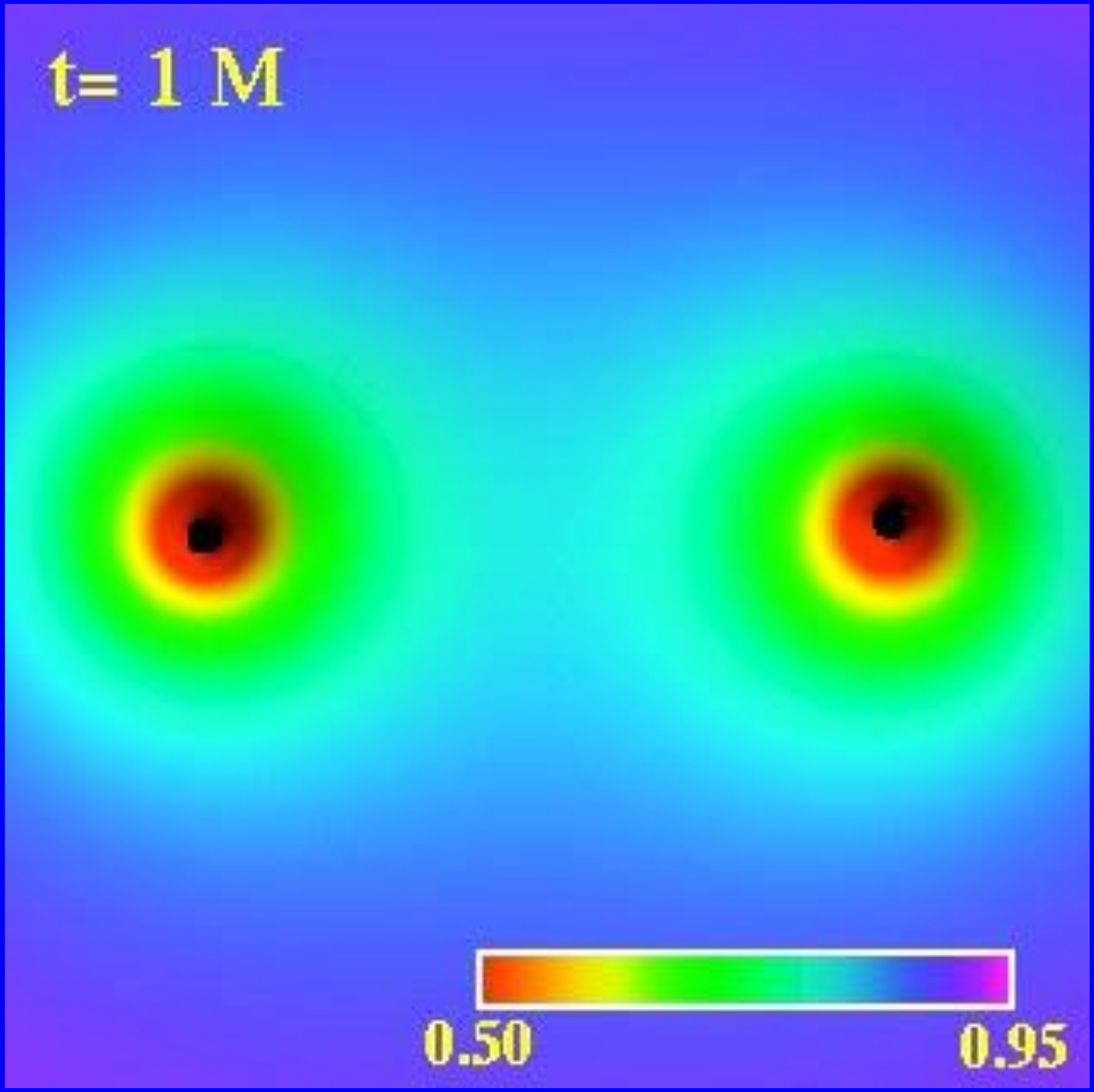
- Uses generalized harmonic approach
- Implements constraint damping: crucial for long-time stability
- Uses excision to avoid singularities
- Maps spatial infinity to finite coordinate distance so that approximate boundary conditions based on large-distance behaviour of gravitational field (asymptotic flatness) are not required
- Uses adaptive mesh refinement (AMR) to efficiently deal with significant range of length-time scales in problem
- Code runs efficiently in parallel on 100’s to 1000’s of processors

Complex code!!
(Generated via symbolic algebra package)

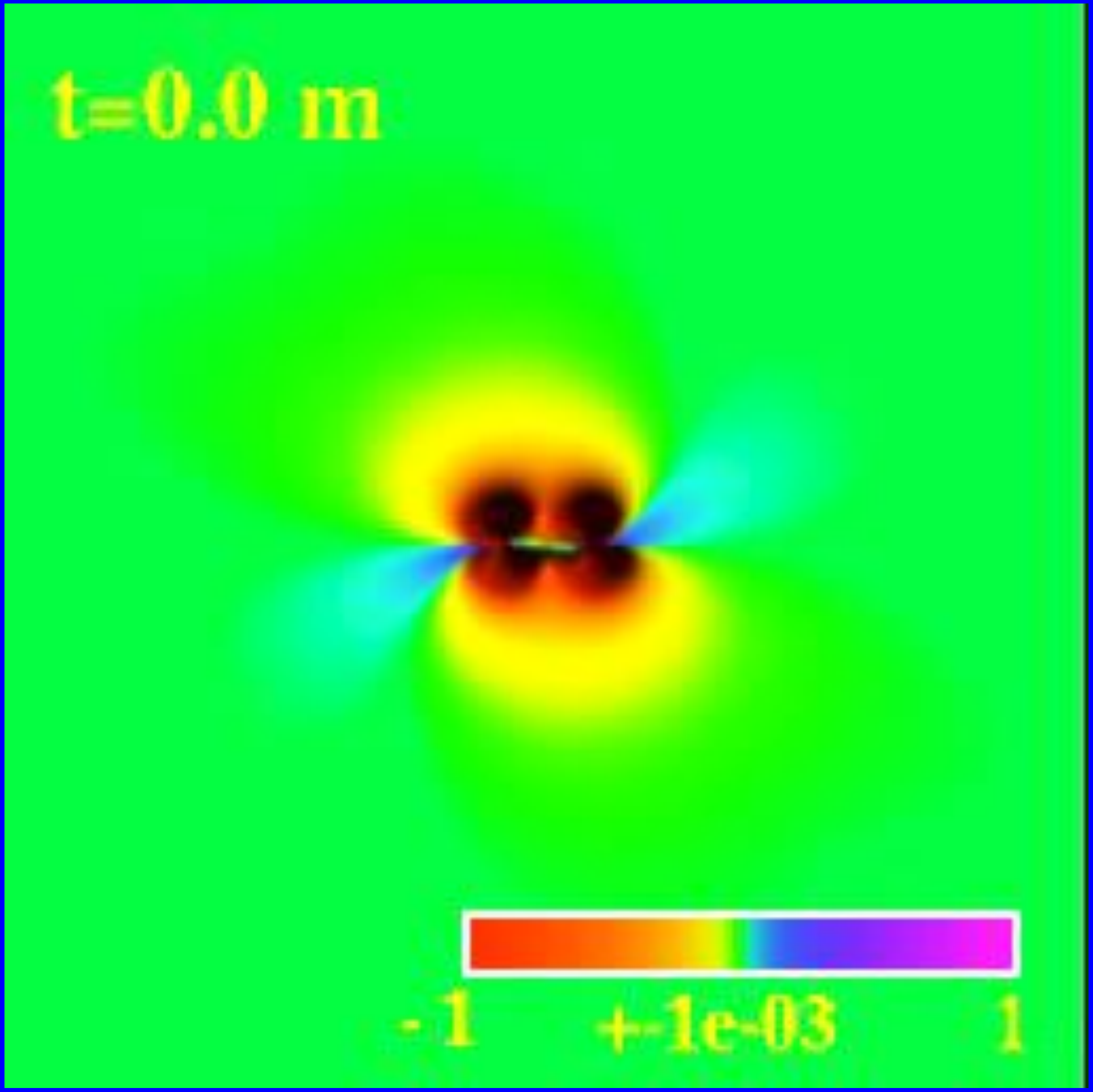
```
 t1109 = t43*csy
 t1126 = gb_zz_t**2
 t1135 = gbu_zz0**2
 t1138 = t43*csz
 t1144 = gb_tt0*t151
 s2 = (t842*gbu_xz0+(2*t451*t459+2*t451*t354+4*t258*t516+t851*gb_xy
#_t-32*t853*t279-32*t118*t274*t119)*gbu_yy0+t880*gbu_yz0+(t534*gb_t
#x_z+t767*gb_ty_z+(t474+t477)*gb_tz_x+t887-t888-32*t889*t279-32*t15
#0*t274*t151)*gbu_zz0+(-2*t897*t28+2*t899*csy)*t12*gb_tt_x+(-2*t905
#*t906+2*t787*t28*csx)*gb_tt_y+t913*gb_tx_t+t793*gb_ty_t+128*t246*p
#hi1_y*t190*phi1_x-4*gb_tt_xy*t12*t77)*gbu_xy0+(2*t608*t268-t766+t9
#27*gb_tx_z-t771+t773+2*t237*t929+t932*gb_xx_t-t775+2*t167*csx*gb_z
#z_t-64*t836*t308)*t940
 s3 = s2+((t624*gb_tx_y+t478*gb_ty_x+t868+t872-t873-32*t853*t308-32
#*t118*t304*t119)*gbu_yy0+t968*gbu_yz0+(2*t608*t598+2*t608*t712+4*t
#289*t929+t962*gb_xz_t-32*t889*t308-32*t150*t304*t151)*gbu_zz0+t989
#*t12*gb_tt_x+(-2*t992*t906+2*t787*t43*csx)*gb_tt_z+t999*gb_tx_t+t7
#93*gb_tz_t+128*t1002*t217*phi1_x-4*gb_tt_xz*t12*t94)*gbu_xz0
 s1 = s3+(2*t451*t516-t1013/4+2*t404*csy*gb_yy_t-16*t118*t325*t119)
#*t1022+((4*t475*t516+2*t451*t641+2*t451*t829-gb_yz_t*gb_yy_t+4*t40
#4*csy*gb_yz_t-32*t118*t357*t119-32*t853*t361)*gbu_yz0+(t1043+(-t53
#1+t533)*gb_ty_z+t1048+t1050-t1052-16*t118*t377*t119-16*t150*t325*t
#151)*gbu_zz0+(-2*t897*t109+2*t899*t1062)*gb_tt_y+t913*gb_ty_t+64*t
#118*t1068-2*gb_tt_yy*t109*t60)*gbu_yy0
 t1154 = s1+(2*t608*t516-t1043+(t531+t533)*gb_ty_z-t1048+t1050+2*t4
#51*t929+t932*gb_yy_t-t1052+2*t404*csy*gb_zz_t-64*t875*t361)*t1089+
#((2*t608*t641+2*t608*t829+4*t475*t929+t962*gb_yz_t-32*t889*t361-32
#*t150*t357*t151)*gbu_zz0+t989*t28*gb_tt_y+(-2*t1107*t77+2*t899*t11
#09)*gb_tt_z+t999*gb_ty_t+t913*gb_tz_t+128*t1002*t333*phi1_y-4*gb_t
#t_yz*t28*t94)*gbu_yz0+(2*t608*t929-t1126/4+2*t564*csz*gb_zz_t-16*t
#150*t377*t151)*t1135+((-2*t984*t141+2*t986*t1138)*gb_tt_z+t999*gb_
#tz_t+64*t150*t1144-2*gb_tt_zz*t141*t60)*gbu_zz0-16*0.3141592653589
#793D1*t5-Hb_t_t
 gb_tt_res = t819+t1154
```

About 100,000 lines of this!! (Much of it to verify correctness of solution)

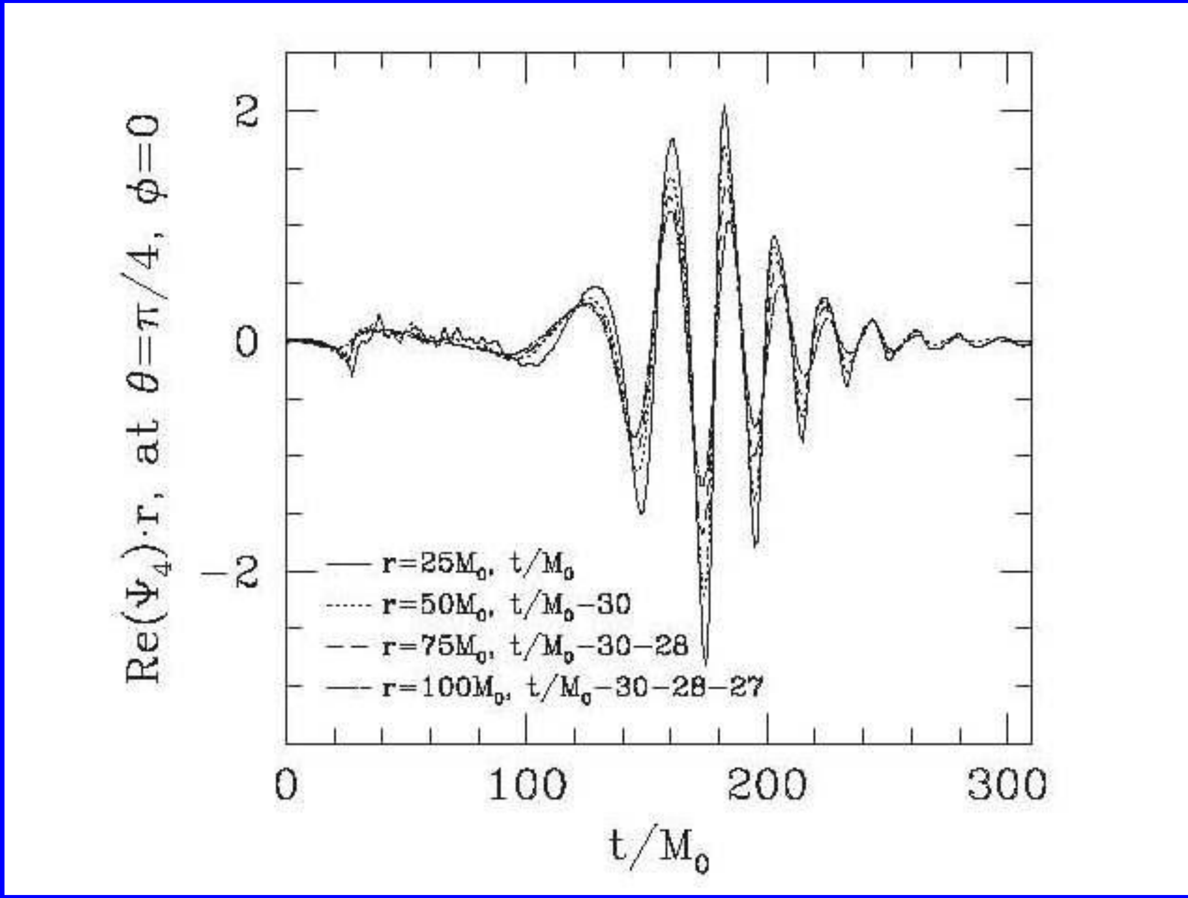
Black Hole Collision
("Lapse function")



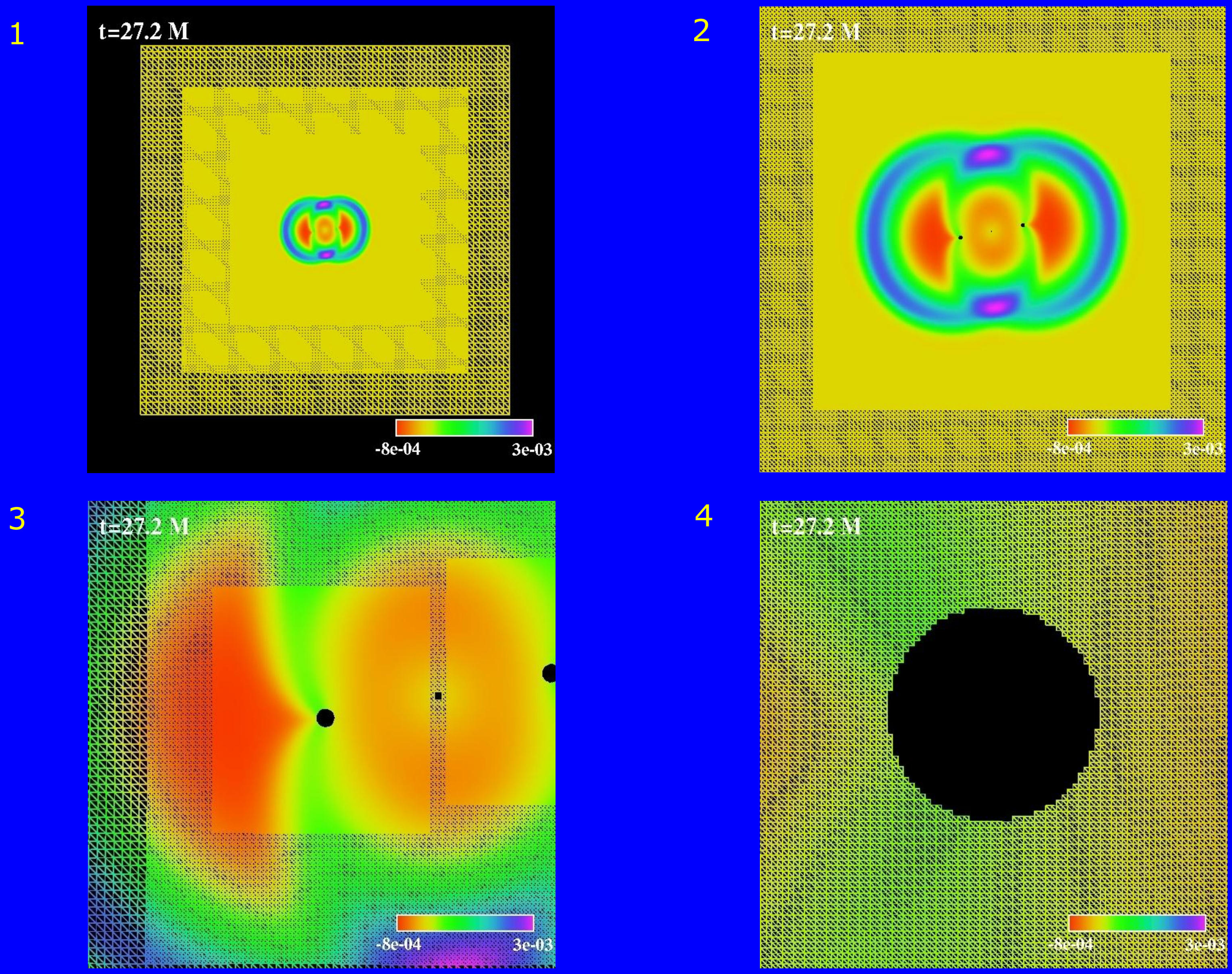
Black Hole Collision
(Gravitational Radiation)



Black Hole Collisions
Computed Gravitational Radiation Waveform



Sample Adaptive Mesh Structure



Looking to the future

- Black hole collisions (inspiraling):
  - Large parameter space (masses, spins) still to be explored
  - Individual calculations still expensive, plagued by “curse of dimensionality”
- Black hole collisions (generic):
  - High energy “scattering” events, although astrophysically implausible, may yield insight into fundamental issues in strong field gravity
  - Tie-in to particle physics (black hole production at the LHC?)

Looking to the future

- Neutron stars
  - NS-NS and NS-BH collisions
  - Realistic equations of state
  - Magnetic fields
- Fundamental issues
  - Continued testing of cosmic censorship hypothesis
  - Detailed nature of singularities inside black holes and in cosmological setting
  - Higher dimensional black objects (black strings, Saturns, …)
  - Clues for “Theory of Everything” (quantum gravity, string theory …)

Opportunities

- Many open problems, relatively few practitioners
- Expertise required (& developed) in many different areas
  - Theoretical general relativity / differential geometry
  - Numerical analysis
  - Software engineering
  - High performance computation
  - Scientific visualization
  - Data analysis
- Developments in computer hardware, software, algorithms will continue to drive field, make it increasingly accessible provided personnel are available

And the Moral is …