Numerical Relativity in the World Year of Physics

2005 CAP Congress
UBC/TRIUMF
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WE-P4-5, IRC 1
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Outline

- Trends (since 1995, say)
- Representative (i.e. the best) results
  - 3D GR-hydrodynamics (collapse, NS collisions; Shibata and collaborators, PRD [2005])
  - 3D GR-vacuum (**BH** collisions, i.e. Minkowski vacuum doesn't count, despite the year!!; Pretorius, unpub.)
- Prognosis

TRENDS: The Good: Hardware

[CFI/ASRA/BCKDF funded HPC infrastructure]

November 1999

vn.physics.ubc.ca
128 x 0.85 GHz PIII, 100 Mbit
Up continuously since 10/98
MTBF of node: 1.9 yrs

glacier.westgrid.ca
1600 x 3.06 GHz P4, Gigiabit
Ranked #54 in Top 500 11/04 (Top in Canada)

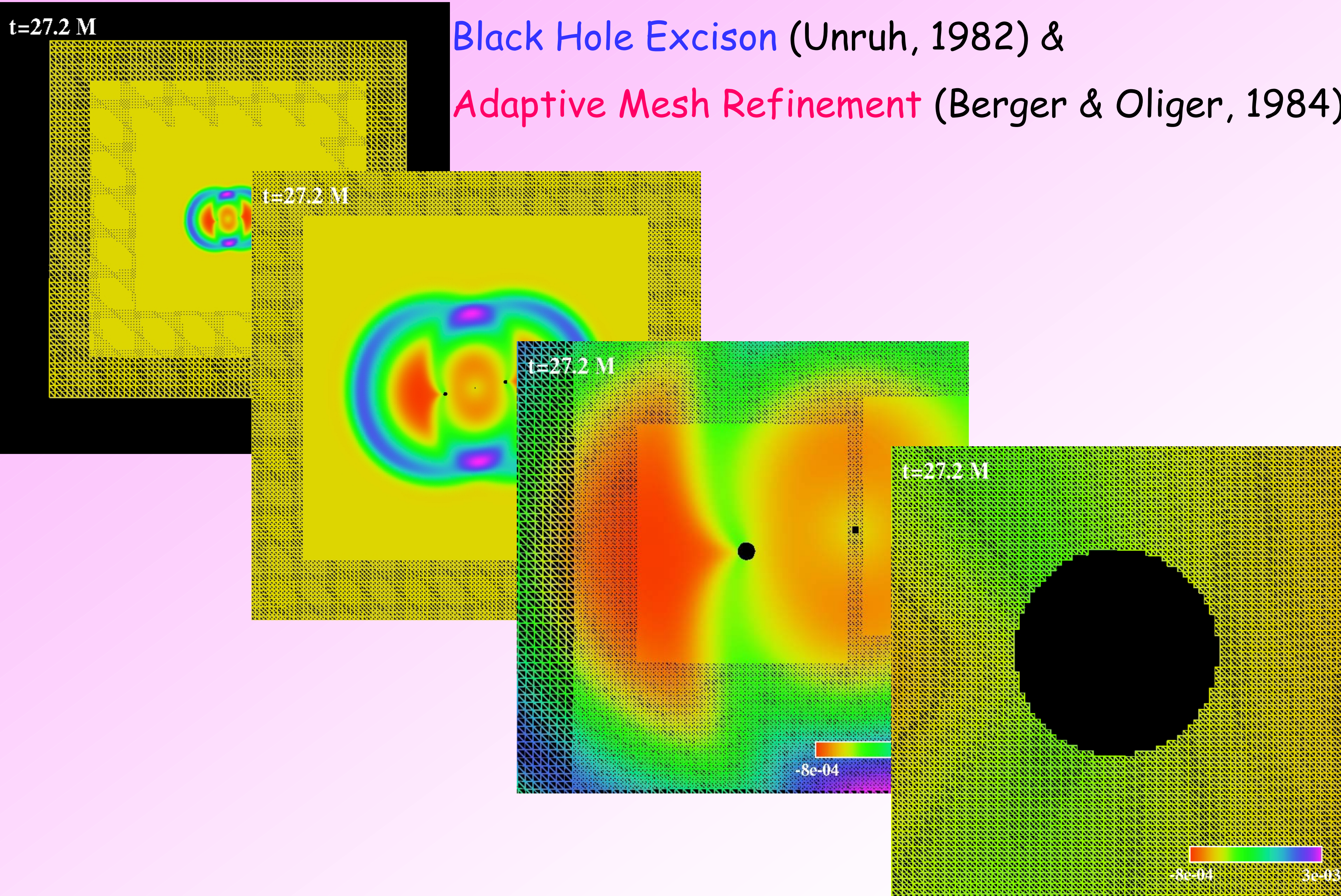
March 2005

vnp4.physics.ubc.ca
110 x 2.4 GHz P4/Xeon, Myrinet
Up continuously since 06/03
MTBF of node: 1.9 yrs

TRENDS: The Good: Ideas & Algorithms

Black Hole Excison (Unruh, 1982) &

Adaptive Mesh Refinement (Berger & Oliger, 1984)



TRENDS: The Good

- Community activity
  - 3D vacuum (largely single BH, very slow progress since 1990 until recent work by Pretorius)
  - 3D matter (in better shape, largely due to lack of horizons for much of evolution, as well as weaker gravitational fields overall relative to BH)
  - Critical phenomena and other "model problems" continue to provide fertile, and arguably the best, training ground for GS, PDFs (Liebling, Hirschmann, Gundlach, Lehner, Neilsen, Pretorius; Hawke others in the wings)

TRENDS: The Good

- Mathematical (incl numerical analytical) maturity
  - Appreciation of importance of hyperbolicity/well-posedness … when solving Einstein equations using free evolution (too many folk to list)
  - Adoption/adaptation of techniques from numerical analysis as a more certain route to stability (LSU group)
  - Successful design and application of constraint dampers for free evolution schemes

TRENDS: The B...

- Community activity
  - 3D vacuum has been focus of roughly 50% or more of the NFS-funded NR effort; to date almost entirely focused on SINGLE BH
  - Excrutiatingly slow, and quite predictable, progress since 1990; no implementation of either of "breakthrough" ideas mentioned above
  - Choice of problems studied, who gets funded, funding level, has had and continues to have little relation to scientific progress; causing resentment among non-N relativists and others

TRENDS: The Ugly

- Places where we probably don't want to go, or should withdraw from while some of the troops are still standing
  - Solving the binary inspiral problem in corotating coordinates
  - Approximately solving Einstein's equations as an INITIAL/BOUNDARY VALUE PROBLEM (IBVP), than as a pure INITIAL VALUE PROBLEM (IVP, Cauchy problem)

TRENDS: The Stark Naked Truth

- Problems we are solving are SIMPLE in specification:
  One page of [tensor] equations, or less;
  In BH-BH case NO PHYSICS OTHER THAN VACUUM GR!!!
  - CAN be "simple" in "implementation"
  - Field dominated, NOT by groups as conventional wisdom would have one believe, but by individuals
    - Fluids (Nakamura, Stark, Evans, Shibata, Miller, …)
    - Vacuum (Bruegmann, Pretorius)
    - Critical Phenomena (…)
  - This fact is being ruthlessly exploited by those keeping their eyes most firmly fixed on the prize (Pretorius, e.g)

Representative Results: 3D GR hydro
(Shibata et al; NS-NS collision; PRD 71:04021 [2005]
3D core collapse; PRD 71:024014 [2005])

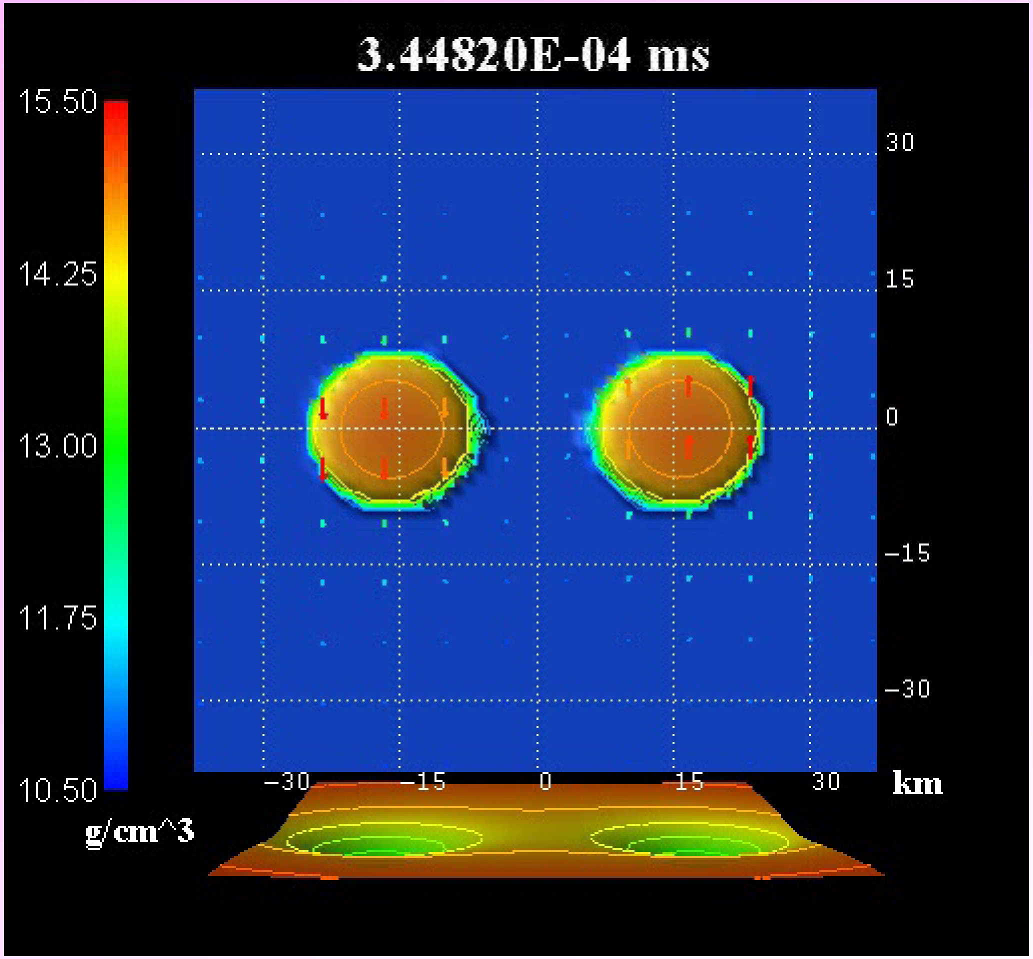
- 3D [x,y,z] (as well as 2D [rho,z], via "Cartoon") solution of Einstein-hydrodynamical equations (fully coupled)
- Key features of approach
  - BSSN formulation of Einstein equations
  - HRSC treatment of hydro; non trivial EOS (multi parameter, "realistic"
  - Single grid, fixed size, but with periodic remap of domain to preserve resolution during collapse, a la Evans)
    - 2D: 2,500 x 2,500 x 40,000: 20 h on 4 procs of FACOM VPP5000, BFM1 (same speed on 8 proc NEC SX6, BFM2)
    - 3D: 440 x 440 x 220 x 15,000: 30 h on 32 processors of BFM1
  - Axisymmetric calcs used in collapse case for hi-res preliminary surveys, identifying candidates likely to display "interesting" behaviour (e.g. instability) in 3D

Inspiral and merger of NS-NS binary

- Initial data
  - Irrotational binary stars in quasiequilibrium circular orbits (?)
  - Separation slightly larger than “innermost orbit” (where Lagrange points appear at the inner edge of the stars)
  - Masses generally chosen in range 1.2 … 1.45 solar

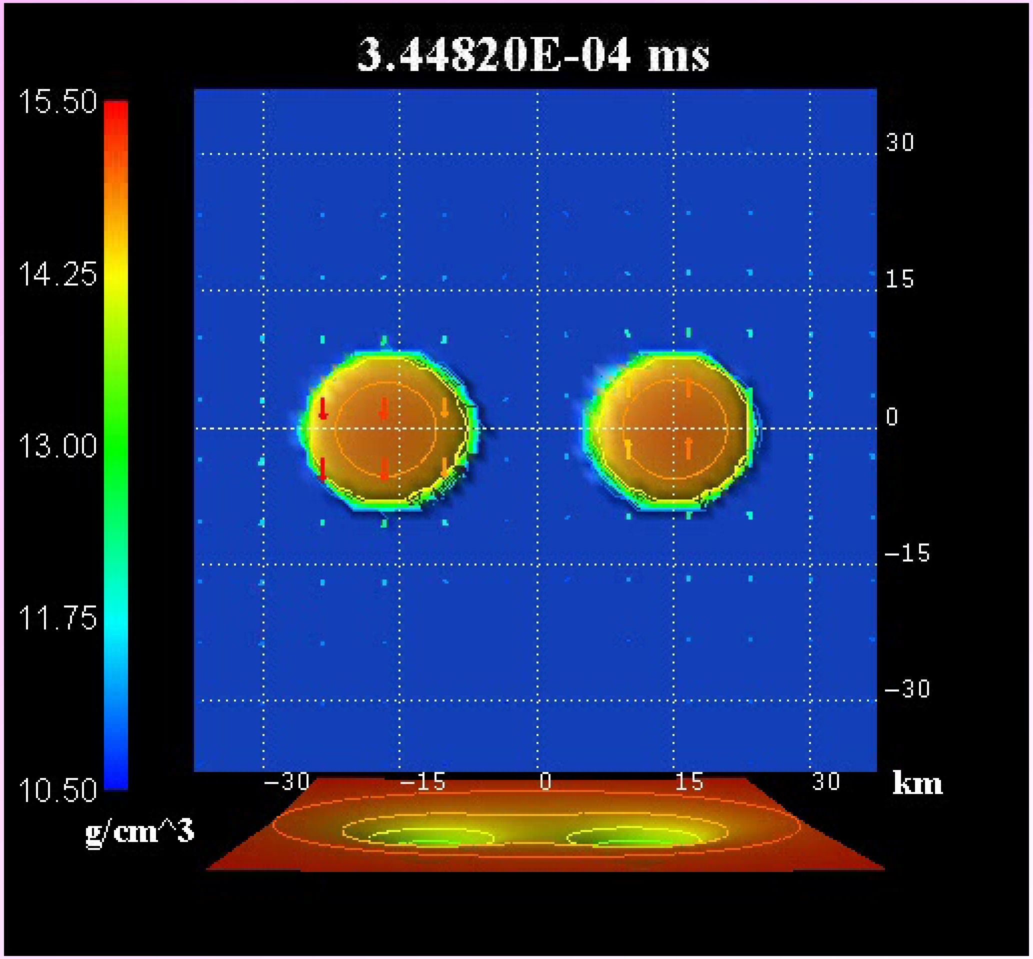
    3 specific cases shown here:
    - 1.30 & 1.30 (equal)
    - 1.25 & 1.35 (unequal)
    - 1.40 & 1.40 (equal)

Masses: 1.30 and 1.30 solar (equal)



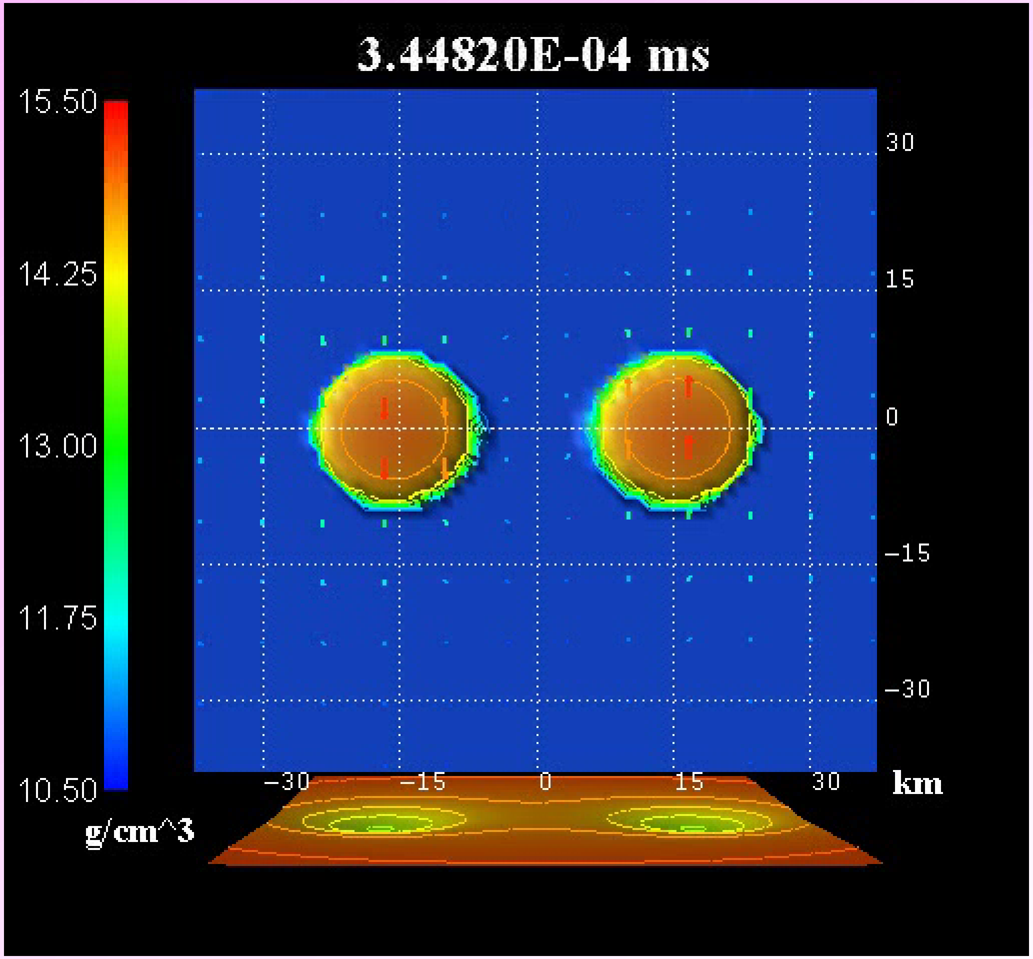
Density contours in x-z plane and lapse visualized.

Masses: 1.25 and 1.35 solar (unequal)



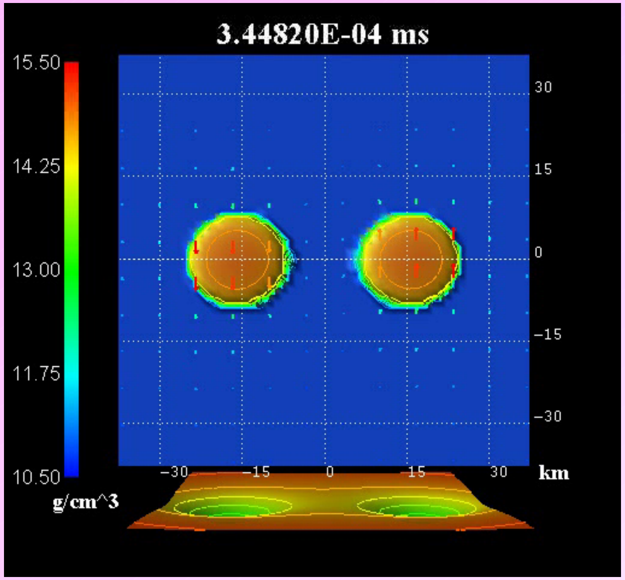
Density contours in x-z plane and lapse visualized.

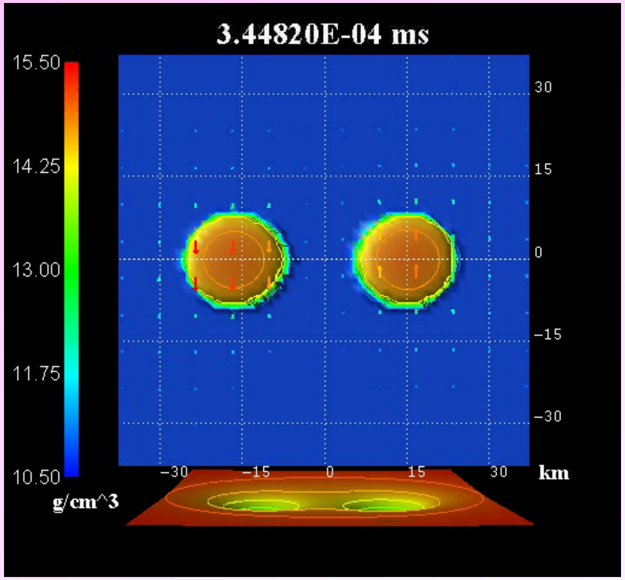
Masses: 1.40 and 1.40 solar (equal)

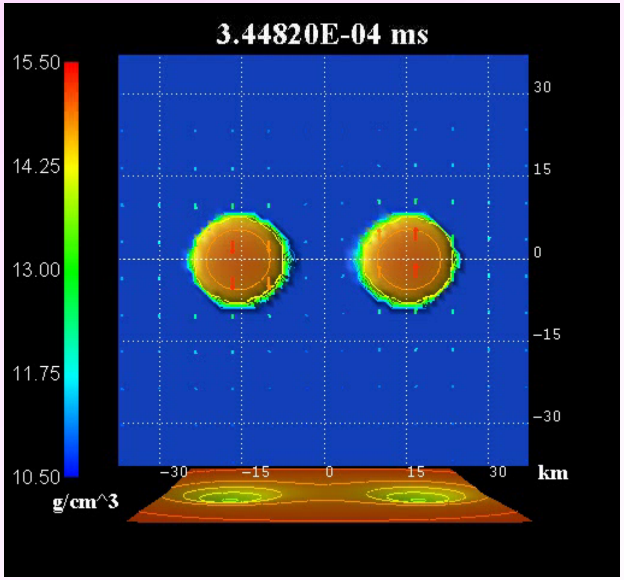


Density contours in x-z plane and lapse visualized.

Left: 1.3 & 1.3 Middle: 1.25 & 1.35 Right: 1.4 & 1.4







Density contours in x-z plane and lapse visualized.

3D core collapse and the development of non-axisymmetric instabilities ("bar modes")

- Initial data
  - Start with axisymmetric code, evolve collapse data (again with realistic equation of state), until configuration reaches some "strong-gravity" point (lapse < 0.8)
  - Then add l=2 perturbation to excite bar mode instability if present

- Key parameter, β, measures how kinetic collapse is, in Newtonian theory, ratio of kinetic & grav. potential energies

$$\beta \equiv \frac{T}{W}$$

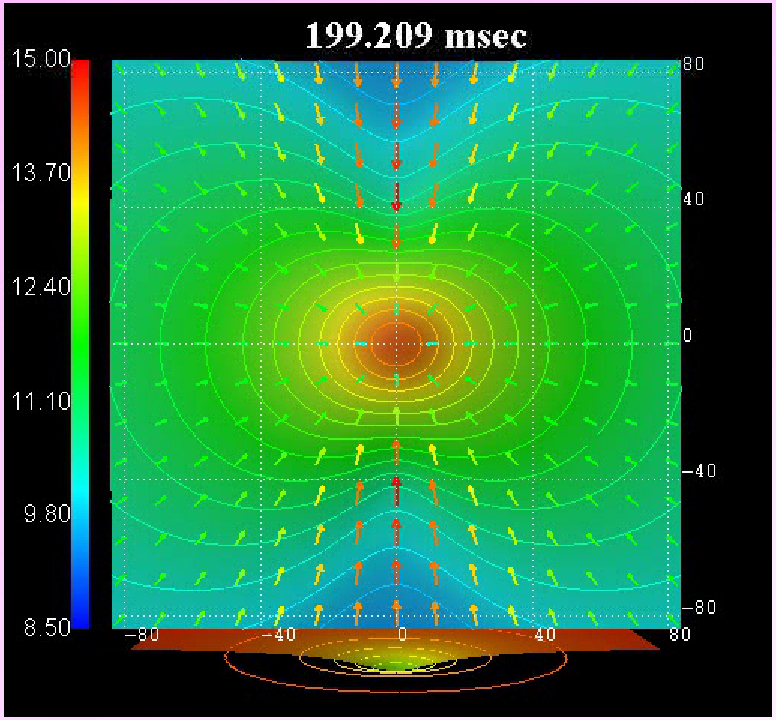
Bar mode onset in stationary (i.e. non collapsing case) when

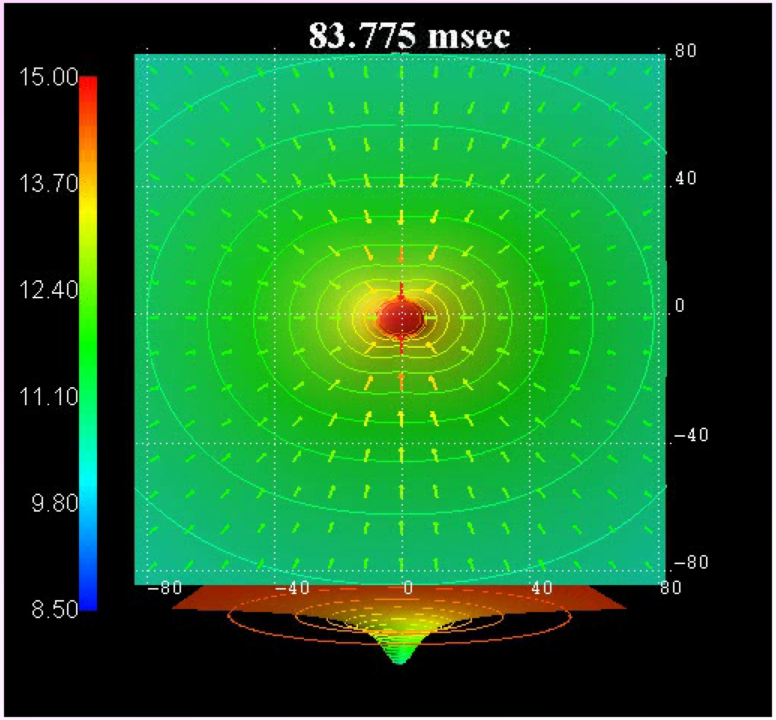
$$\beta > \beta_c \approx 0.27$$

3D core collapse and the development of non-axisymmetric instabilities ("bar modes")

- EOS: Γ_1, Γ_2 : different Γ above/below nuclear density
- Cores shown in the 2.5 – 3.0 solar range
- Initial betas of order 0.001, maximum achieved, order 0.3; those configs getting there tend to be oscillating stars above nuclear density
- Total gravitational radiation emitted as high as 0.03% of total mass, much higher than in axisymmetric collapse

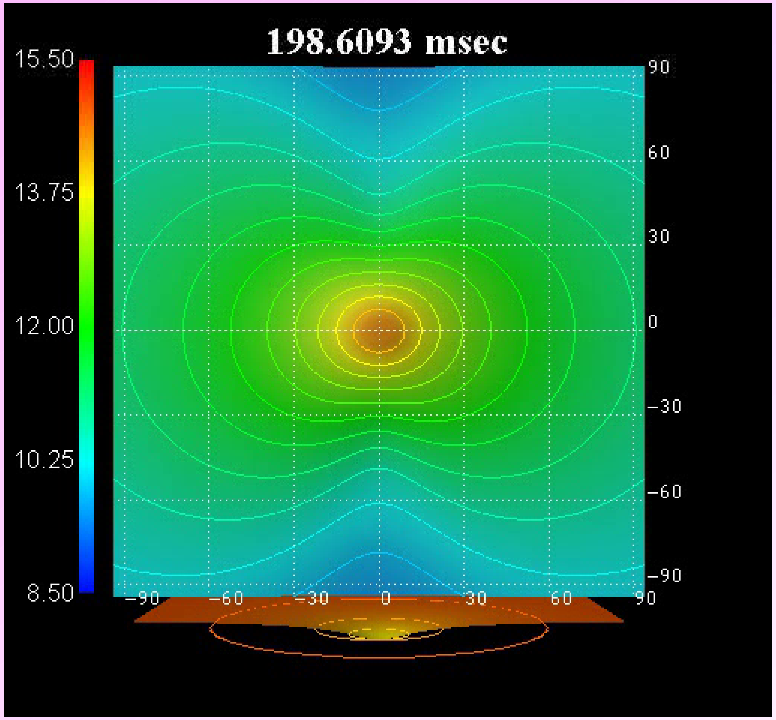
Core collapse to NS
(matter contours in x-z plane; evolution of lapse)

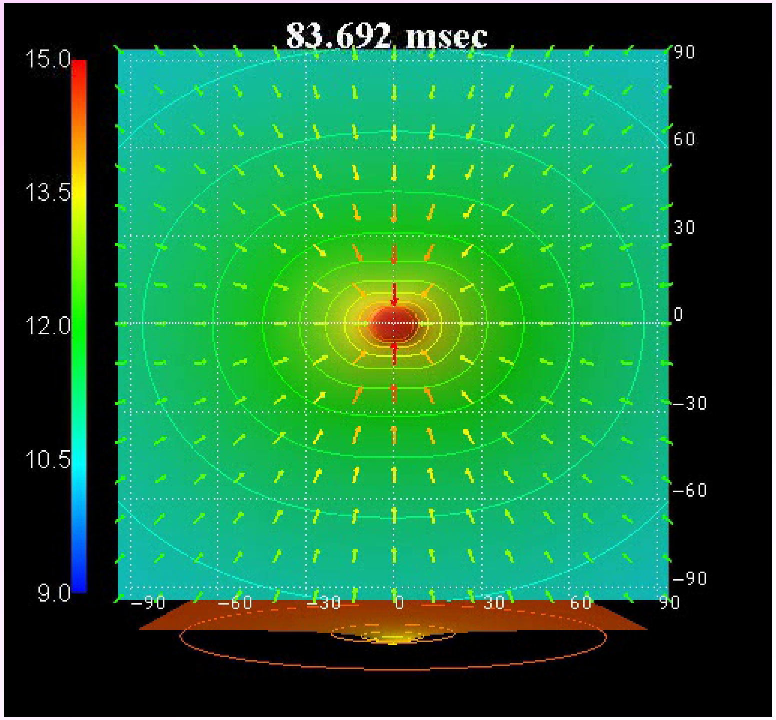




Density contours in x-z plane and lapse visualized.

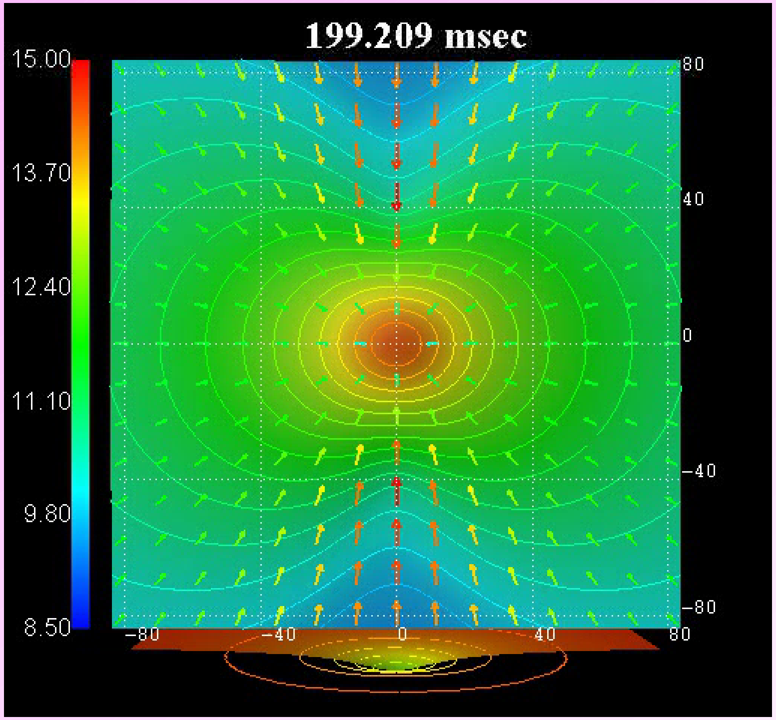
Core collapse to BH
(matter contours in x-z plane; evolution of lapse)

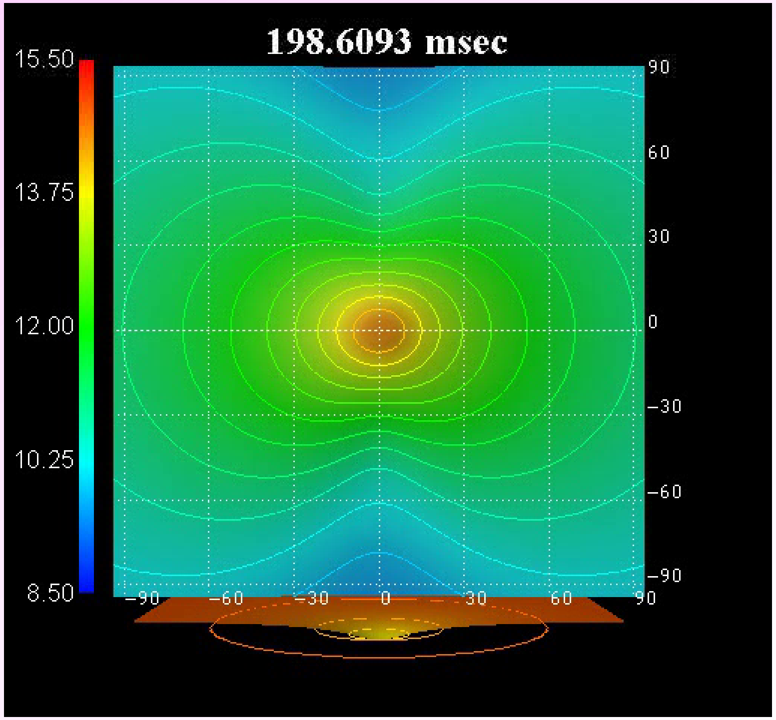




Density contours in x-z plane and lapse visualized.

Comparison of collapse to NS (left) and BH (right)





Density contours in x-z plane and lapse visualized.

Representative Results: 3D vaccum
(Pretorius, unpublished [2005])

- Key features of approach (in development for about 3 yrs)
  - "ad hoc"; ignored much "conventional wisdom" (often when CW had no empirical basis)
  - Arguably only fundamentals retained from 30 years of cumulative experience in numerical relativity:

1. Geometrodynamics is a useful concept (**Dirac**, **Wheeler** …)
2. Pay attention to constraints (**Dewitt,** …)

Pretorius's New Code: Key Features

- GENERALIZED harmonic coordinates
- Second-order-in-time formulation and direct discretization thereof
- $O(h^2)$ finite differences with iterative, point-wise, Newton-Gauss-Seidel to solve implicit equations
- Kreiss-Oliger dissipation for damping high frequency solution components (stability)
- Spatial compactification
- Implements black hole excision
- Full Berger and Oliger adaptive mesh refinement
- Highly efficient parallel infrastructure (almost perfect scaling to hundreds of processors, no reason can't continue to thousands)
- Symbolic manipulation crucial for code generation

Pretorius' Generalized Harmonic Code

[Class. Quant. Grav. 22, 425, 2005,
following Garfinkle, PRD, 65:044029, 2002]

- Adds "source functions" to RHS of harmonic condition

$$\nabla^{\alpha}\nabla_{\alpha}x^{\mu} \equiv \frac{1}{\sqrt{-g}}\partial_{\alpha}\left(\sqrt{-g}g^{\alpha\mu}\right) = H^{\mu}$$

- Substitute gradient of above into field equations, treat source functions as INDEPENDENT functions: retain key attractive feature (vis a vis solution as a Cauchy problem) of harmonic coordinates

$$g^{\gamma\delta}g_{\alpha\beta,\gamma\delta} + \ldots = 0$$

Principal part of continuum evolution equations for metric components is just a wave operator

Pretorius' Generalized Harmonic Code

- Constraints:

$$C^{\mu} \equiv H^{\mu} - \nabla^{\alpha}\nabla_{\alpha}x^{\mu} = 0$$

Can NOT be imposed continuously if source functions are to be viewed/treated as independent of the metric functions

Choosing source functions from consideration of behaviour of 3+1 kinematical variables

$$ds^2 = -\alpha^2 dt^2 + h_{ij}\left(dx^i + \beta^i dt\right)\left(dx^j + \beta^j dt\right)$$

$$H \cdot n \equiv H_\mu n^\mu = -n^\mu \partial_\mu \ln\alpha - K$$

$$\perp H^i \equiv H_\mu h^{i\mu} = \frac{1}{\alpha} n^\mu \partial_\mu \beta^i + h^{ij}\partial_j \ln\alpha - \bar{\Gamma}^i_{jk} h^{jk}$$

$$\partial_t \alpha = -\alpha^2 H \cdot n + \ldots$$

$$\partial_t \beta^i = \alpha^2 \perp H^i + \ldots$$

Choosing source functions from consideration of behaviour of 3+1 kinematical variables

- Can thus use source functions to drive 3+1 kinematical vbls to desired values

- Example: Pretorius has found that all of the following slicing conditions help counteract the “collapse of the lapse” that generically accompanies strong field evolution in “pure” harmonic coordinates

$$H_t = \xi \frac{\alpha - 1}{\alpha^n}$$

$$\partial_t H_t = \xi \partial_t \left(\frac{\alpha - 1}{\alpha^n} \right)$$

$$\nabla^\mu \nabla_\mu H_t = -\xi \frac{\alpha - 1}{\alpha^n} - \zeta \partial_t H_t$$

Constraint Damping

[Brodbeck et al, J Math Phys, 40, 909 (1999); Gundlach et al, gr-qc/0504114]

- Modify Einstein/harmonic equation via

$$g^{\alpha\beta} g_{\mu\nu,\alpha\beta} + \ldots + \kappa \left(n_\mu C_\nu + n_\nu C_\mu - g_{\mu\nu} n^\alpha C_\alpha \right) = 0$$

where

$$C^\mu \equiv H^\mu - \nabla^\alpha \nabla_\alpha x^\mu$$
$$n_\mu \equiv -\alpha \nabla_\mu t$$

- Gundlach et al have shown that for all positive κ, (to be chosen empirically in general), all non-DC contraint-violations are damped for linear perturbations about Minkowski

Merger of eccentric binary systems
(Pretorius, work in progress)

- Initial data
  - Generated from prompt collapse of balls of massless scalar field, boosted towards each other
  - Spatial metric and time derivative conformally flat
  - Slice harmonic (gives initial lapse and time derivative of conformal factor)
  - Constraints solved for conformal factor, shift vector components

- Pros and cons to the approach, but point is that it serves to generate orbiting black holes

Merger of eccentric binary systems

- Coordinate conditions

$$\nabla^{\mu}\nabla_{\mu}H_t = -\xi\frac{\alpha-1}{\alpha^n} - \zeta\partial_t H_t$$

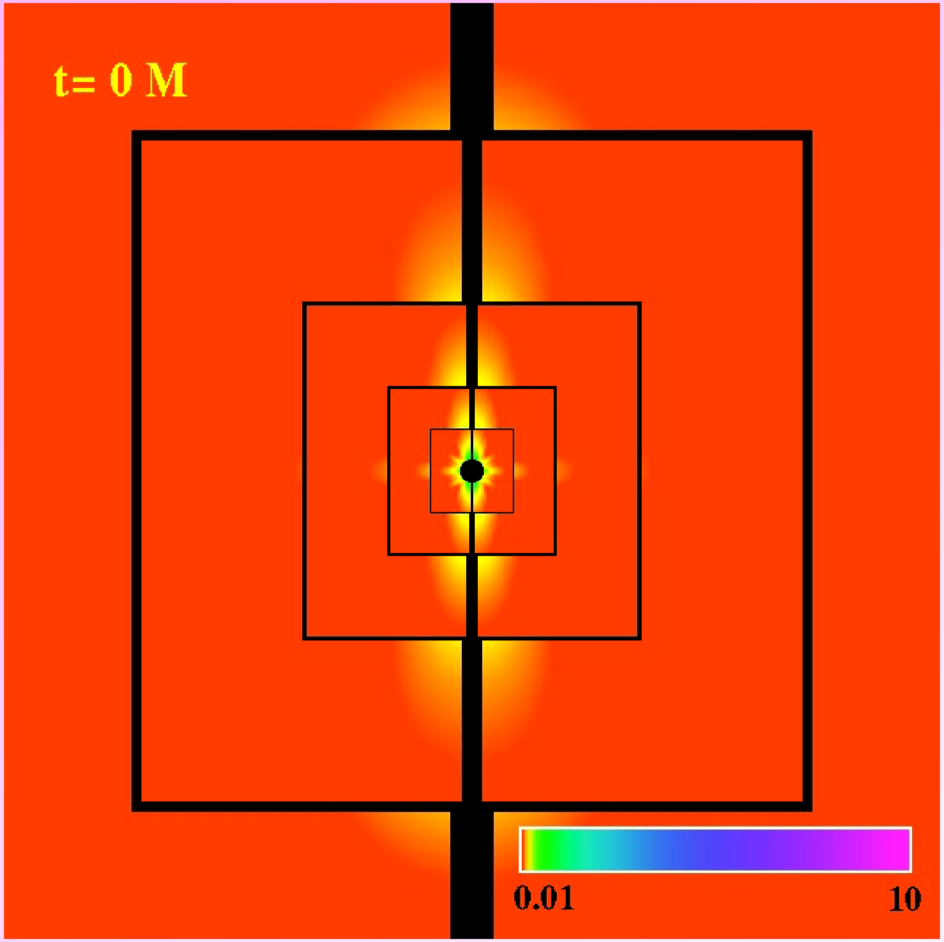
$$H_i = 0$$

$$\xi \sim 6/M, \quad \zeta \sim 1/M, \quad n = 5$$

  - Strictly speaking, not spatially harmonic, which is defined in terms of "contravariant components" of source fcns

- Constraint damping coefficient: $\kappa \sim 1/M$

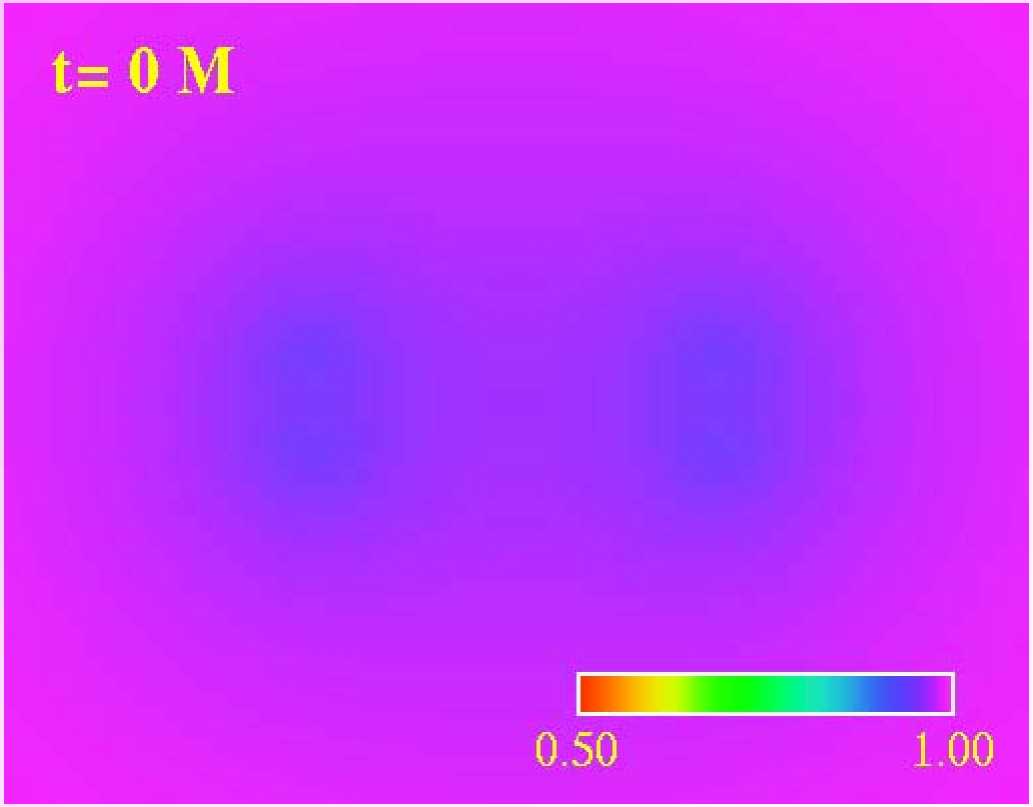
Effect of constraint damping



Constraint violation visualized

- Axisymmetric simulation of single Schwarzschild hole
- Left/right calculations identical except that constraint damping is used in right case
- Note that without constraint damping, code blows up on a few dynamical times

Representative Results: GR vacuum
(Merger of eccentric system; Pretorius, unpub. [2005])



Lapse function visualized

Representative Results: GR vacuum
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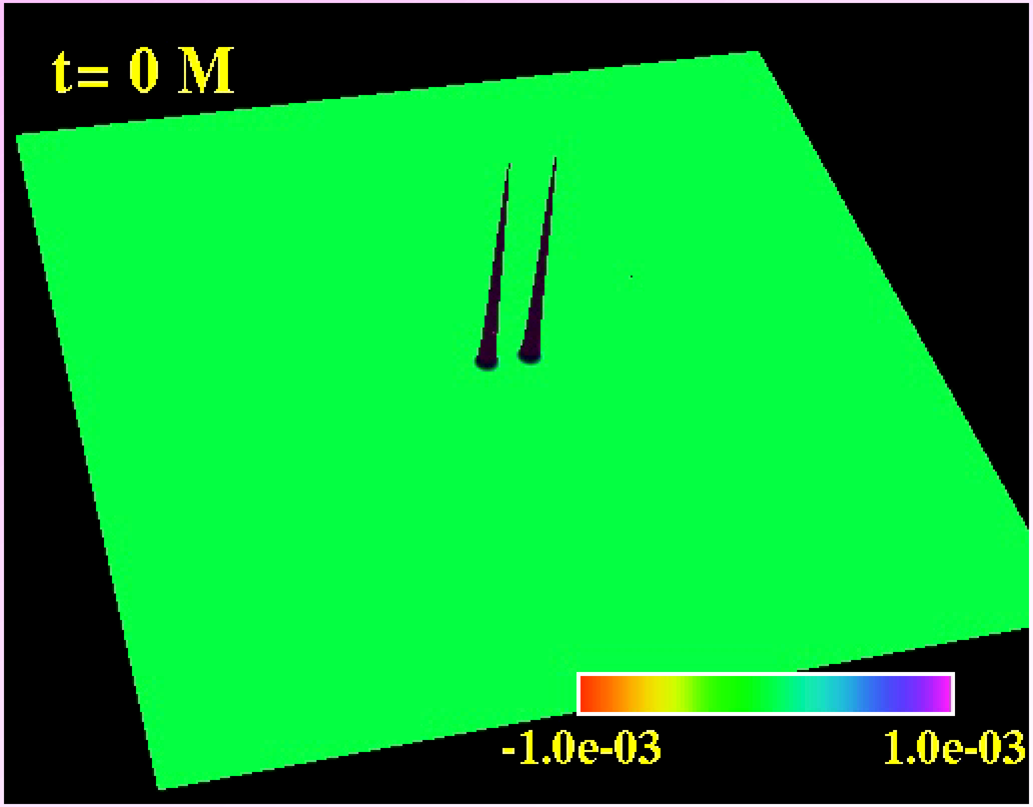
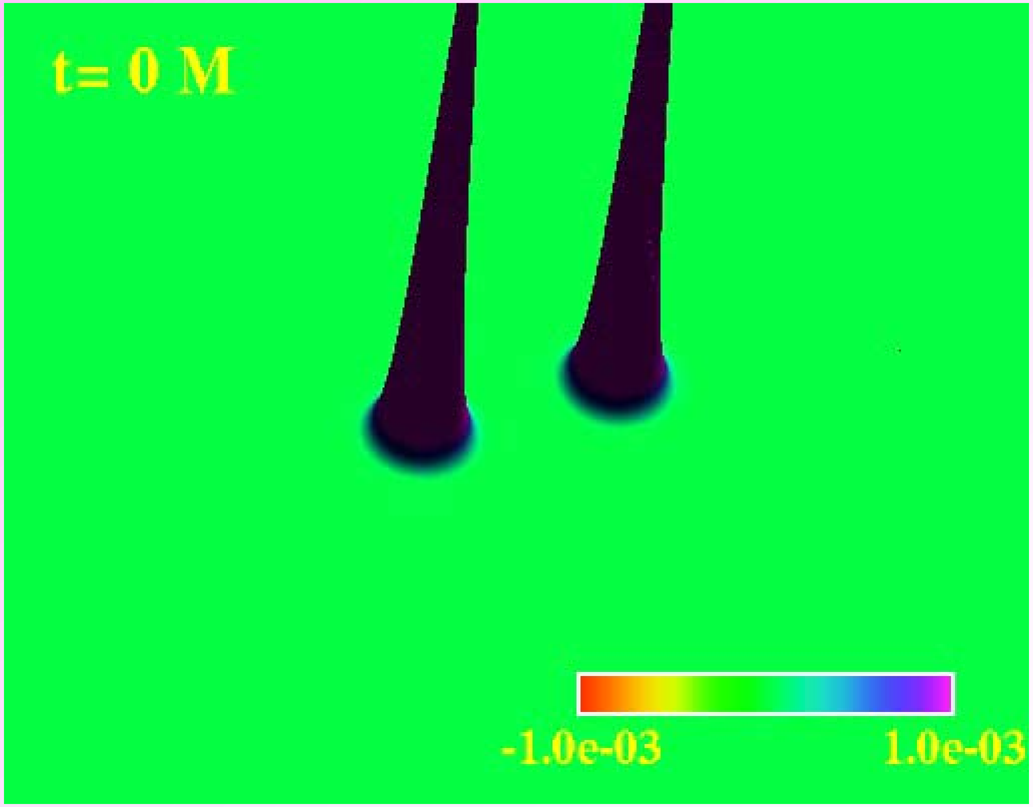


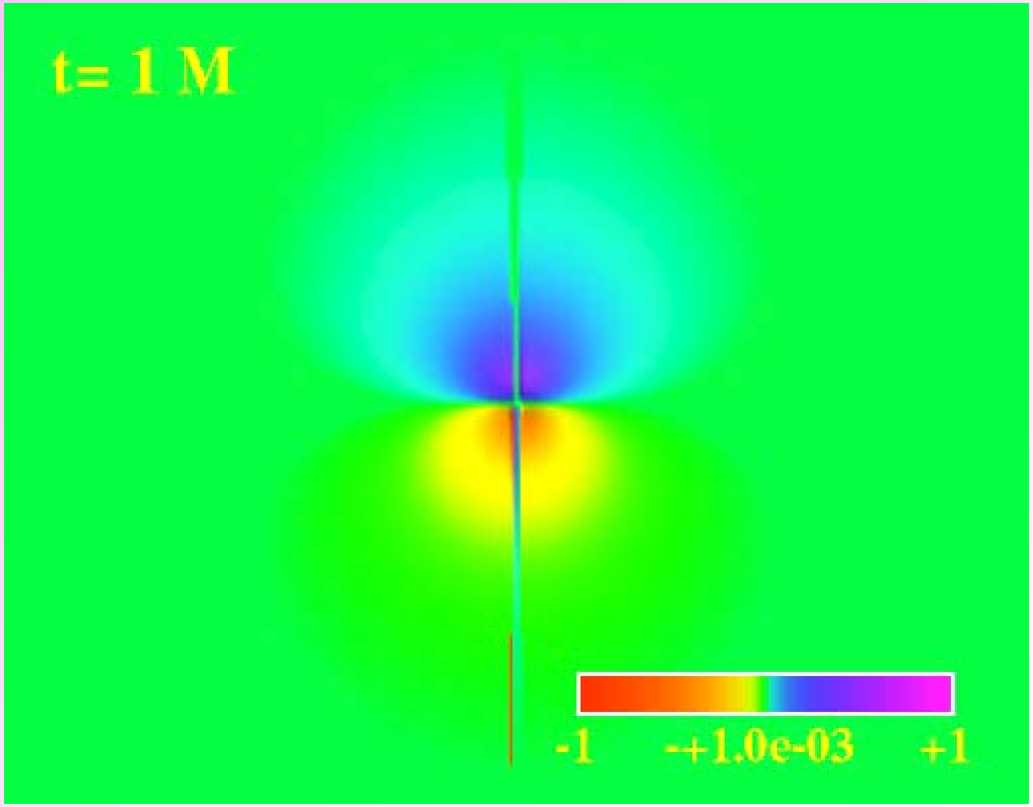
Scalar field visualized (computational/compactified coords.)

Representative Results: GR vacuum
(Merger of eccentric system; Pretorius, unpub. [2005])



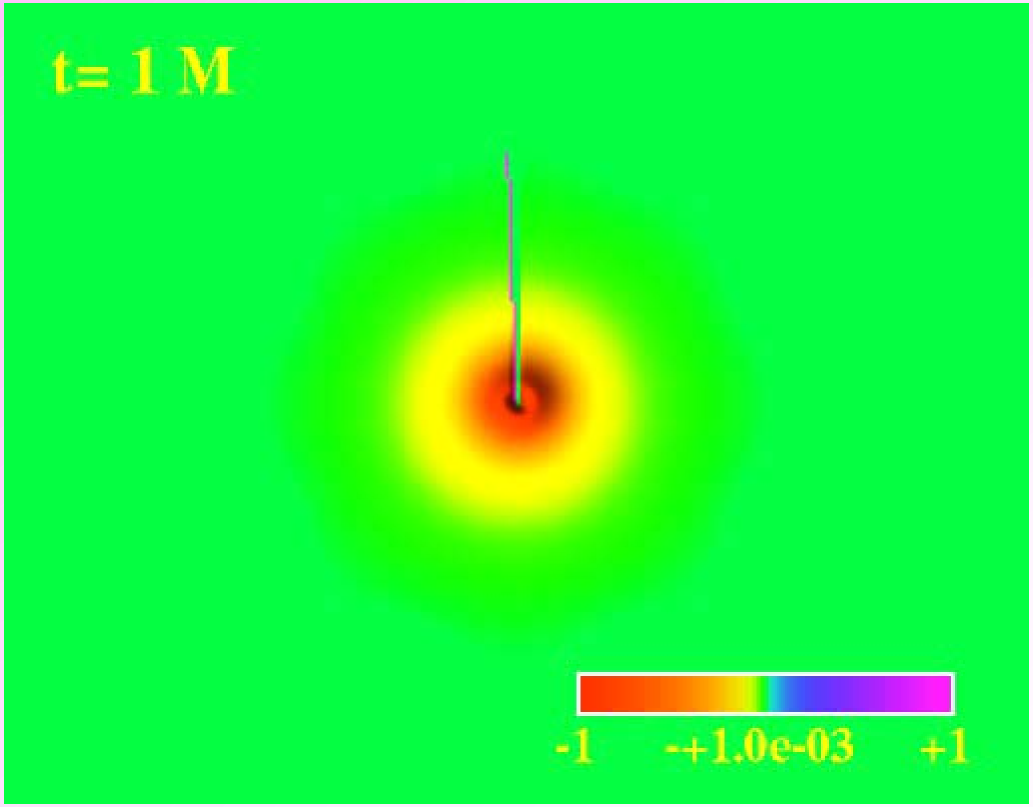
Scalar field visualized (uncompactified coords.)

Representative Results: GR vacuum
(Merger of eccentric system; Pretorius, unpub. [2005])



"Gravitational radiation" visualized

Representative Results: GR vacuum
(Merger of eccentric system; Pretorius, unpub. [2005])



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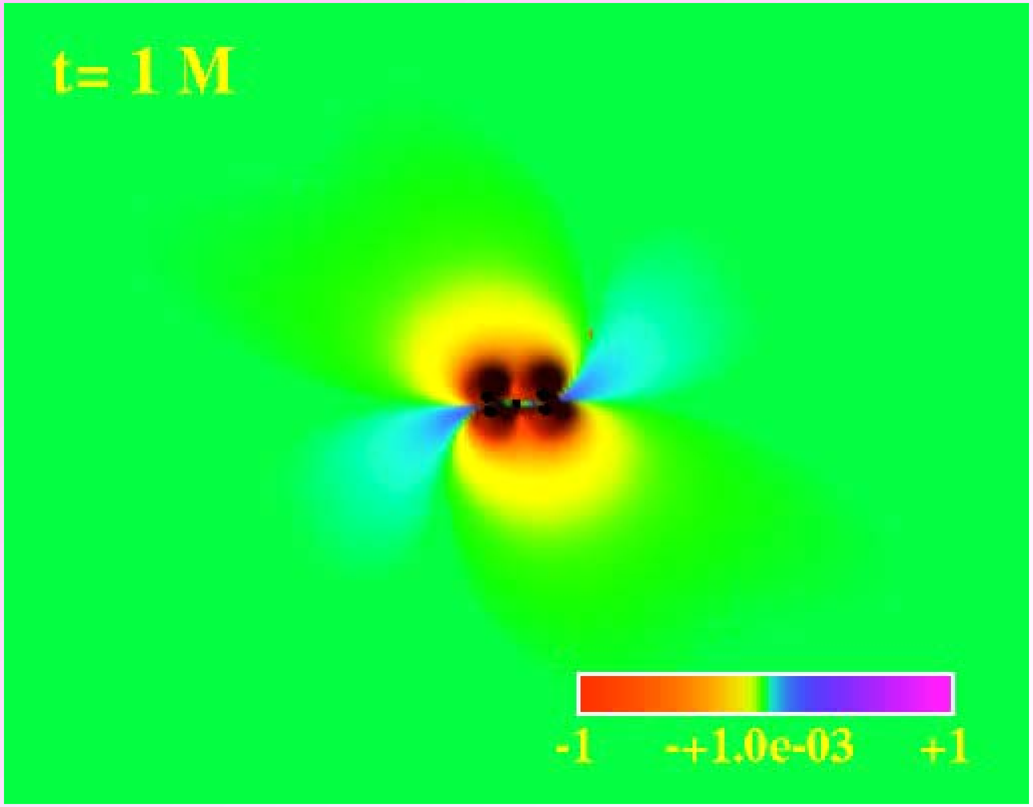

"Gravitational radiation" visualized

Computation vital statistics

- Base grid resolution: 48 x 48 x 48
  - 9 levels of 2:1 mesh refinement
    - Effective finest grid 12288 x 12288 x 12288
- Calculation similar to that shown
  - ~ 60,000 time steps on finest level
  - CPU time: about 70,000 CPU hours (8 CPU years)
    - Started on 48 processors of our local P4/Myrinet cluster
    - Continues of 128 nodes of WestGrid P4/gig cluster
  - Memory usage: ~ 20 GB total max
  - Disk usage: ~ 0.5 TB with infrequent output!

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  - Have scaling issues to deal with, particularly with low-order difference approximations in 3 (or more!) spatial dimensions; but there are obvious things to be tried

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  - Have scaling issues to deal with, particularly with low-order difference approximations in 3 (or more!) spatial dimensions; but there are obvious things to be tried
- STILL LOTS TO DO AND LEARN IN AXISYMMETRY AND EVEN SPHERICAL SYMMETRY!!

APS Metropolis Award Winners
(for best dissertation in computational physics)

1999	LUIS LEHNER
2000	Michael Falk
2001	John Pask
2002	Nadia Lapusta
2003	FRANS PRETORIUS
2004	Joerg Rottler
2005	HARALD PFEIFFER