Ultrarelativistic Particle Collisions

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We present results from numerical solution of the Einstein field equations describing the head-on collision of two solitons boosted to ultrarelativistic energies. We show, for the first time, that at sufficiently high energies the collision leads to black hole formation, consistent with hoop-conjecture arguments. This implies that the nonlinear gravitational interaction between the kinetic energy of the solitons causes gravitational collapse, and that arguments for black hole formation in super-Planck scale particle collisions are robust.

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I. Introduction.—Using ultrarelativistic scattering of particles to probe the nature of the fundamental forces has a long tradition in modern physics. One reason why is the de Broglie relation, stating that the characteristic wavelength of a particle is inversely related to its momentum, and consequently probing short-range interactions between particles requires large momenta. In any conventional setting gravity is an irrelevant force in such interactions. However, in general relativity all forms of energy, including momentum, gravitate, and thus at sufficiently high energies one would expect gravity to become important. The current paradigm suggests that this will happen when center of mass energies approach the Planck scale, and for collisions with energies sufficiently above this, that black holes will be formed [1].

The four dimensional Planck energy E_p is $\approx 10^{19}$ GeV, wholly out of reach of terrestrial experiments, and as far as known is not reached by any astrophysical process, barring the big bang or the unobservable regions inside black holes. However, if there are more than four dimensions, intriguing scenarios have been suggested where the true Plank scale is very different from what is then just an effective four-dimensional Planck scale [2]. A "natural" choice for the true Planck energy is the electroweak scale of ~TeV, as this would solve the hierarchy problem. If this were the case, and the paradigm of black hole formation is correct, this would imply that the Large Hadron Collider (LHC) will produce black holes, and that black holes are formed in Earth's atmosphere by cosmic rays [3]. Existing experimental bounds on the Planck energy in this context are at around 1 TeV [4].

However, one potential problem with the above scenario, even before one considers issues regarding the existence of extra dimensions, physics near the Planck regime, etc., is whether in classical general relativity the generic outcome of ultrarelativistic two "particle" scattering is a black hole for small impact parameters. There have so far been no solutions to the field equations demonstrating this, and as we will outline next, the evidence usually presented for the case of black hole formation is based on a set of conjectures and the use of limiting-case solutions of dubious applicability.

The main argument for black hole formation is a variant of Thorne's hoop conjecture [5]: if a total amount of matter and energy E is compressed into a spherical region such that a hoop of proper circumference $2\pi R$ completely encloses the matter in all directions, a black hole will form if the corresponding Schwarzschild radius $R_s = 2GE/c^4$ is greater than R, where G is Newton's constant and c the speed of light. To apply this to the collision of two classical spherical solitons, each with rest mass m_0 and traveling toward each other with speed v in the center of mass frame, let $E = 2\gamma m_0 c^2$, the total energy of the system, where $\gamma = 1/\sqrt{1 - v^2/c^2}$. The largest radius R to be enclosed by the hoop is the rest-frame radius R_0, as the Lorentz contraction only flattens the particles in the direction of propagation. The hoop conjecture then says black holes will form if $\gamma \gtrsim c^4 R_0/4Gm_0$.

The above argument is purely classical. Quantum mechanics enters with the assumption that the argument still holds for the collision of fundamental particles, now taking R_0 to be the de Broglie wavelength hc/E of the particle, where h is Planck's constant. Dropping constants of $O(1)$, the criteria for black hole formation is then $E \gtrsim (hc^5/G)^{1/2}$, the Planck energy. However, our goal in this Letter is only to address the soundness of the classical hoop-conjecture argument; if it fails, there is no reason to expect a full quantum version to hold.

It is not obvious that the hoop conjecture is applicable in all situations. Consider a single particle boosted beyond the Planck energy. Since the boosted particle's spacetime is a coordinate transformation of its rest-frame geometry, there is clearly no black hole formation. As trivial as this example may seem, it is still insightful, as it illustrates that not all forms of energy gravitate in the same way. Here, kinetic energy, unlike rest mass energy, does not produce spacetime curvature, yet both forms of energy contribute to the mass of the spacetime, as measured for instance by the ADM mass [6]. The kinetic energy dominates the rest mass energy by orders of magnitude, and for black hole formation to be a generic outcome the particular nature of the particles and nongravitational interactions between them cannot play a role. Therefore, it must be the nonlinear interaction between opposing streams of gravitational kinetic energy that causes a black hole to form, in the process converting kinetic energy to rest mass and gravitational wave energy.

What is often quoted as evidence for black hole formation comes from the study of the collision of two infinitely boosted "particles," described by the Aichelburg-Sexl (AS) metric [7]. The AS solution is obtained by taking a Schwarzschild black hole of mass m, applying a Lorentz boost γ, and then taking the limits $\gamma \to \infty$ and $m \to 0$, so that the product $E = \gamma m$ remains finite. The result is a gravitational "shock wave," where the nontrivial geometry is confined to a two-dimensional plane traveling at the speed of light, with Minkowski spacetime on either side. Two such solutions, moving in opposite directions, can be superimposed to give the precollision geometry of the spacetime (see [8] for an insightful description). Though the geometry is not known to the future of the collision, at the moment of collision a trapped surface can be found [9]. Assuming cosmic censorship, this would be an example of black hole formation in an ultrarelativistic collision.

There are several aspects of the infinite boost construction that should give one pause as to its applicability to a large-yet-finite γ collision of massive particles. The AS limit is not asymptotically flat, and the algebraic type of the metric has changed from Petrov type D (two distinct null eigenvectors of the Weyl tensor) to Petrov type N (one null eigenvector). This latter point can be thought of as the gravitational field changing from a Coulomb-like to a pure gravitational wave field. The AS metric is also not a good description to the geometry of a finite boosted particle on the shock surface; one is then left with the uninsightful conclusion that the description is good sufficiently far from the particle that its metric is Minkowski. It has also been argued that black hole formation is due to the strong focusing of geodesics off the AS shock wave [10]. However, there is no dynamics in this description (neither in the superposed AS metrics for that matter), and it is difficult to imagine how the geodesic structure can capture what is a highly nonlinear and dynamical interaction between gravitational energy.

To test the hypothesis that black holes form in high energy collisions of particles, we numerically solve the Einstein field equations coupled to matter that permits stable, self-gravitating soliton solutions—these are our model particles. The particular soliton we use is a boson star [11]. One motivation for choosing this model was from earlier studies of low velocity, head-on collisions of boson stars, which suggested that as the velocity increases, gravity appears to "weaken," and the boson stars pass through each other exhibiting a nonrelativistic solitonic interference pattern [12]. In particular, the magnitude of the bosonic matter field developed interference fringes of wavelength $\lambda \propto 1/P$, with P the momentum of each boson star, which is exactly the relationship observed during the collision of Bose-Einstein condensates bound via Newtonian gravity. This model therefore seems perfect to address the genericity requirement for black hole formation at super-Planck scale collisions, in that the self-interaction of the matter will not bias the outcome toward black hole formation (as, for example, using black holes as model particles would).

II. Methodology.—We solve the Einstein equations, $R_{ab} - g_{ab}/2R = 8\pi T_{ab}$, using a variant of the generalized harmonic formalism [13] with constraint damping [14] as described in [15]. In this formalism, the spacetime coordinates satisfy a wave (harmonic) condition $\Box x^a = H^a$, with the H^a encoding the coordinate (gauge) degrees of freedom. Experimentation with coordinate choices led us to develop what we term a damped harmonic condition, which can be written $H^a = \xi[n^a - \bar{n}^a]$, where ξ is a constant, n^a is the timelike unit normal to the $t =$ const slices, and $\bar{n}^a$ is another timelike unit normal field which is to be chosen so that the resulting coordinate system is nonsingular. Such an approach was independently introduced in [16], where the choice $\bar{n}^a = (\partial/\partial t)^a/\alpha + \log(\alpha/\sqrt{h})n^a$, with α the lapse function and h the determinant of the spatial metric, was proposed. We use a variant of this condition that transitions to $H^a = 0$ shortly after the collision.

The matter is a minimally coupled complex scalar field ϕ with mass parameter m, equation of motion $\Box\phi = m^2\phi$, and stress tensor $T_{ab} = 2\nabla_{[a}\phi\nabla_{b]}\bar{\phi} - g_{ab}(\nabla_c\phi\nabla^c\bar{\phi} + m^2\phi\bar{\phi})$, where $\bar{\phi}$ is the complex conjugate of ϕ. For initial data we superimpose two boosted boson stars following a procedure analogous to that for binary black holes presented in [17]. This construction by itself does not fully satisfy the constraint equations. However, the further apart the boson stars are at $t = 0$, the smaller the error is, and we have performed tests that indicate for the initial separations here the error is sufficiently small to not qualitatively affect the conclusions.

Each boson star is identical with a central scalar field amplitude ϕ_0 chosen so that the maximum compactness $2M(r)/r$ of each star is $\approx 1/20$, where the mass aspect $M(r)$ approaches the ADM mass M_{ADM} as $r \to \infty$. We subsequently scale all units to $M_0 \equiv 2M_{\mathrm{ADM}}$. We choose an initial coordinate separation between the boson stars in the center of mass (simulation) frame of $d_0 = 250M_0$, and give each boson star a boost of γ. Thus, with this compactness, the hoop-conjecture estimate of the black hole formation threshold is $\gamma_h \approx 10$.

III. Results.—Our key result is the simple answer "yes" to the question of do ultrarelativistic boson star collisions lead to black hole formation in classical general relativity?

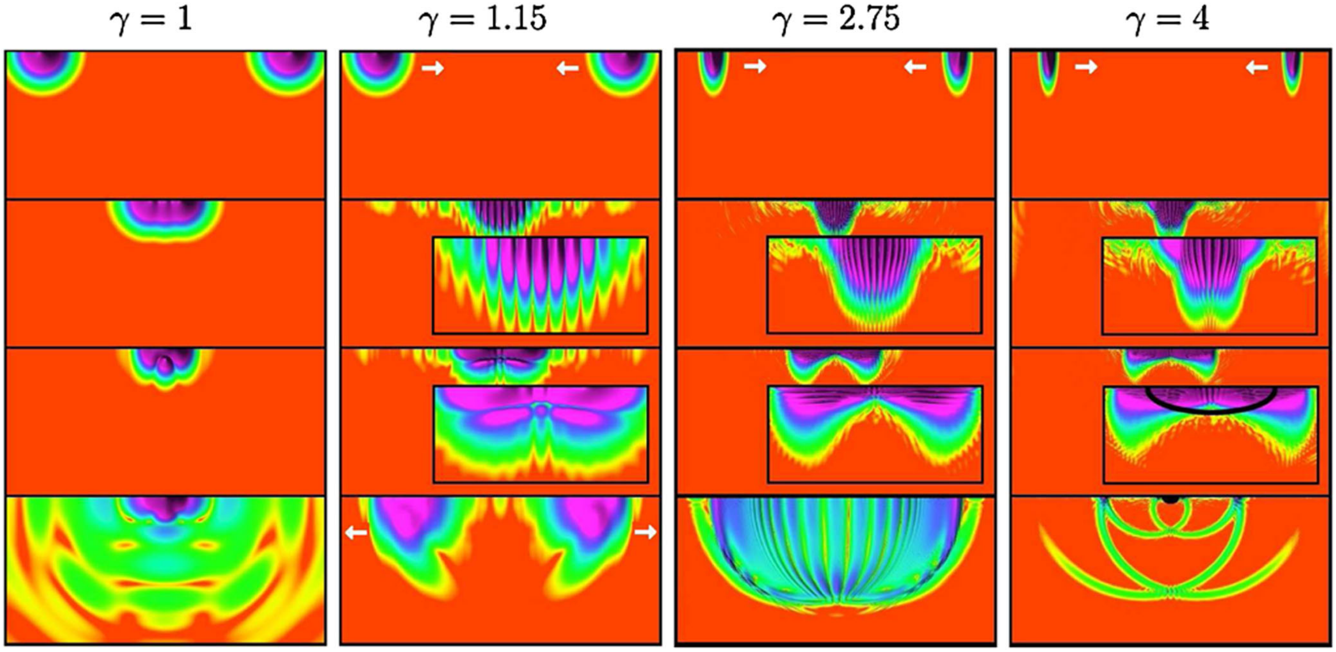

FIG. 1 (color online). Magnitude $|\phi|$ of the scalar field from 4 different simulations, in 4 panels (left to right). The 4 subpanels within each panel depict $|\phi|$ at different times as follows (top to bottom): (1) $t/M_0 = 0$, (2) a time at which the boson stars first completely overlap, (3) a short time later when $|\phi|$ reaches a first local maximum due to gravitational focusing, (4) a late time after the collision. The axis of symmetry is coincident with the top edge of each subpanel. The insets, where present, are zoom-ins of the central interaction regions. For the $\gamma = 4$ case, a black hole forms near the time of subpanel 3—the black line in the corresponding inset shows the shape of the apparent horizon then, and the black semicircle in subpanel 4 is the excised region inside the black hole.

Despite the objections we noted to the use of the hoop conjecture in this scenario, it therefore does appear that the argument captures the essential physics of high speed soliton collisions. We find black hole formation at $\gamma_c = 2.9 \pm 10\%$, less than a third that predicted by the hoop conjecture. Of course, the latter is an order of magnitude estimate, and the numerical value of the threshold may depend on the particular soliton model. Note that the maximum compactness of single, stable boson stars is ~ 0.25 (see, e.g., [18]), though it is not immediately apparent whether this is of relevance here.

Figure 1 shows snapshots of the scalar field for several boost parameters at key times. For the collision beginning at rest ($\gamma = 1$), a single perturbed boson star forms, undergoing large oscillations that slowly damp via the emission of scalar radiation. For larger boosts, the initial boson star interaction exhibits the usual nongravitational interference pattern, but shortly afterward there is some compression of the stars due to gravity. For the modest boost of $\gamma = 1.15$, though the stars are perturbed by the compression, they pass through each other. Approaching the threshold with $\gamma = 2.75$, the compression is much greater, and though the boson stars pass through each other the perturbation is strong enough to cause them to "explode." I.e., though the bulk of the momentum in the scalar field is concentrated in two fronts propagating outward along the axis, a non-negligible component appears to move outward in spherical shells emanating from two focal points, corresponding to the locations of maximum compression seen in subpanel 3.

For the $\gamma = 4$ case the interaction is similar to $\gamma = 2.75$ until apparent horizon formation; this is consistent with the intuition that in this regime the "matter does not matter"—it is the gravitational energy determining the dynamics, and here the scalar field is merely a tracer of the underlying geometry. After black hole formation, the resultant evolution is strikingly different. Most of the scalar field falls into the black hole, though a small fraction escapes. Similar to the $\gamma = 2.75$ case, at late times (subpanel 4) the matter that escapes can be traced back to appear to have originated as two pulses on the axis at the locations of maximum compression (subpanel 3). However, now a piece of each outward moving wave front gets trapped in the black hole. The wave front remains connected, and with time this causes the concentric outward propagating arc patterns seen in subpanel 4.

Figure 2 depicts the gravitational wave emission, as measured by the Newman-Penrose scalar Ψ_4, for the two higher γ cases of Fig. 1. Note, however, that the damped harmonic coordinates cause a strong "distortion" of the metric near the solitons, which eventually propagates outward with the gravitational wave. This prevents a clean interpretation of Ψ_4, defined here with a tetrad aligned with the coordinate basis vectors, as representing the gravitational wave signal. With this caveat in mind, there is an interesting feature suggested by Fig. 2. The wavelength in

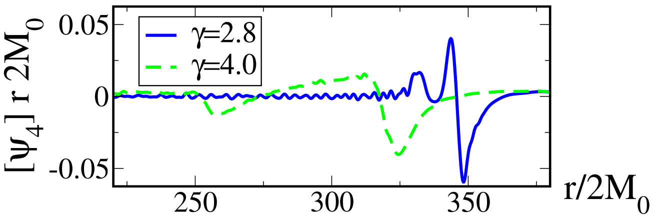

FIG. 2 (color online). Ψ_4 at $t = 540M_0$ on the plane passing through the collision point and orthogonal to the axis.

the black hole formation case is consistent with the wave being associated with the dominant quasinormal ringdown mode of the black hole. However, in the subthreshold case the characteristic wavelength is quite a bit shorter, and there seems to be a trend of smaller length-scale features developing the closer to threshold we tune, which will continue if the threshold exhibits type II critical behavior [19]. As it relates to particle collisions, this suggests black hole formation may not be the ''end of short-distance physics,'' but that the low energy relationship equating small distances to large momenta ceases to be valid, and probing physics at smaller distances requires fine-tuning the interaction energy.

IV. Conclusions.— We presented numerical results from a first study of the ultrarelativistic collision of solitons within general relativity. The goal was to test if at sufficiently high energy gravity dominates the interaction, leading to black hole formation. We found that, for this class of soliton, the conjecture is true, and the threshold of black hole formation occurs at a boost γ_c approximately $1/3$ that predicted by Thorne's hoop conjecture. Interestingly, a factor of $\sim 1/3$ also arises in calculations of trapped-surface formation in the collision of null sources following an S-matrix approach to the scattering problem [20]. With $\gamma_c = 2.9 \pm 10\%$ the ratio of kinetic to rest mass energy is $\approx 2{:}1$; we believe this is sufficiently large to make a compelling case that black hole formation is generic in ultrarelativistic particle collisions, regardless of the internal structure of the particles. Thus the arguments that super-Planck scale particle collisions lead to black hole formation are robust, and furthermore using black holes as the model particle to study the gravitational aspects of the interaction at these energies [8,9,21] is valid. Our results also suggest that when gravity becomes a strong player in the interaction at scales slightly below the Planck scale, the de Broglie relationship equating smaller distances to larger energies may cease to hold. Of course, here the nature of quantum gravity will be crucial, and may provide its own cutoff to short-distance physics, though gravity will not play the role of the censor.

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