Gravitational Collapse in Axisymmetry

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Motivation

Why 2D, rather than 3D?

- computationally less expensive than 3D, so we can solve problems with sufficient resolution on modest computer systems
- simpler equations, thus we can hope for quicker turn-around time developing new ideas

Why axisymmetry?

- system still general enough to study a wide range of interesting problems, of both theoretical and astrophysical interest, including:
  - gravitational collapse and critical phenomena
  - head-on black hole collisions
  - cosmic censorship and the hoop conjecture

Description of our Approach

We are using the [(2+1)+1] formalism, developed by Maeda, Sasaki, Nakamura and Miyama (1979). It is an application of the ADM space + time decomposition to a dimensionally reduced manifold, obtained by projecting out the Killing vector as devised by Geroch (1970).

The result can be thought of as 3 dimensional gravity with source 'matter' fields, that describe the effects of angular momentum in 4D. These effective matter fields are the magnitude of the Killing vector :

$$\lambda = \xi^a \xi_a$$

and its twist:

$$\omega_a = \epsilon_{abcd} \xi^b \nabla^c \xi^d$$

For the 3D space-time, we have decided to use cylindrical coordinates, and a conformally flat spatial 2-metric:

$$\begin{aligned} {}^{(3)}ds^2 &= \left[-\alpha^2 + \psi^4 \left(\beta^{\rho 2} + \beta^{z2}\right)\right] dt^2 \\ &+ 2\psi^4 \left(\beta^\rho d\rho + \beta^z dz\right) dt + \psi^4 \left(d\rho^2 + dz^2\right) \end{aligned}$$

As of yet, we are not modeling rotation, so $\omega_a = 0$, and we define λ as follows:

$$\lambda = \psi^4 e^{2\sigma}$$

All variables are functions of ρ, z and t.

For a matter source, we have a real, minimally coupled, massless scalar field $\phi(\rho, z, t)$; though in the near future we intend to add electromagnetism.

Boundary conditions

- outer boundary
  - assume $1/r$ fall-off for ψ,α,β^ρ and β^z; outgoing radiation conditions for the scalar field and σ.
- inner boundary
  - enforce regularity on the axis $\rho = 0$. This amounts to enforcing the even or odd character of each function on the axis.

Numerical Code

- Fully constrained evolution, with maximal slicing $\rightarrow$ 4 elliptic equations for ψ,α,β^ρ and β^z, solved for using an FAS Multigrid algorithm.

- For the remaining hyperbolic-like equations, we use a 2-time level, Crank-Nicholson scheme, implemented in RNPL. We apply numerical dissipation (via Kreiss-Oliger), which is essential for stability.

- The code is 2nd order accurate, and consistent (modulo outer-boundary effects).

Current Work: Excision and AMR

Excision

To deal with the singularities that occur when black holes form, we are implementing Apparent Horizon excision (Unruh).

The idea is that, assuming cosmic censorship, the behavior of spacetime and 'realistic' matter fields inside an apparent horizon cannot effect the physics outside of it, so we can remove the interior region from the computational domain.

Our excision strategy is as follows:

- specify a smooth family of level surfaces $f(\rho, z)$, and excise within the region $f < f_0$. f_0 is chosen to lie a certain distance *within* the AH.

- use one-sided differencing for 'evolved' variables along the surface of excision, and add dissipation.

- Dirichlet boundary conditions set for α, β^ρ and β^z:

  - for scalar field dominated, single black hole spacetimes it has been sufficient to freeze in the values of these variables at the time of excision
  - for head-on black hole collisions, we manipulate β^z to force the merger to occur in coordinate space
  - Brill wave collapse will require a more dynamical set of conditions, such as area-locking.

Solutions with excision exhibit a slight inconsistency that may require modification of the simplistic Dirichlet style conditions.

AMR

- AMR will be *essential* to study critical phenomena in our coordinate system.

- AMR will be of significant value in other problems, such as BH collisions: it will allow us to push the outer boundaries far enough away to obtain 'cleaner' waveforms, while resolving the BH's in sufficient detail to avoid dissipation and smoothness problems along the surface of excision.

- Help reveal new physics?

- Current status of AMR:

  - initial data solution, i.e. MG on an adaptive hierarchy, is working well.

  - evolution is still very much 'work in progress'; the most pressing issue being the incorporation of elliptic equations into Berger and Oliger style mesh-refinement, which is tailored to the solution of hyperbolic systems.

Conclusion - goals for the ‘near’ future

- critical phenomena
- consistent excision
- cosmic censorship investigation
- include angular momentum
- include electromagnetism